Derivation of Cochannel and Adjacent Channel Reuse Ratio Distributions in DCA Cellular Systems

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***Abstract*—This paper presents the first closed-form analysis of cochannel and adjacent channel reuse ratio distributions in dynamic channel assignment (DCA) cellular systems and compares the results with conventional fixed channel assignment (FCA) cellular systems. Computer simulations show that DCA systems exhibit significantly closer cochannel and adjacent channel frequency reuse than FCA systems for a significant proportion of terminals. Mathematical analysis is used to show that this is a fundamental consequence of DCA channel pooling and that the resultant interference distributions cannot be obtained using conventional cellular engineering techniques. The closed-form expressions derived establish a theoretical lower limit to channel reuse in DCA cellular systems.**

***Index Terms*—Cellular coverage, cellular radio, microcells, radio interference, reuse ratios.**

I. Introduction

IN CONVENTIONAL cellular radio communications systems ("macrocell" systems), the available radio channels are partitioned into C channel sets (C is called the "cluster size"), and each transmitter is allocated the use of one of these channel sets [1], [2]. This is called fixed channel assignment (FCA). Once the system grows beyond C cells, the channel sets are reused in such a way as to not cause excessive interference to existing cells.

The channel reuse ratio in a cellular system is defined as the ratio of the distance d between cells using related channels to the cell radius r. If a pair of terminals in two cells are using the same channel, the ratio d/r is called the cochannel reuse ratio (CRR). If a pair of terminals in the two cells are using immediately adjacent channels, the ratio d/r is called the adjacent channel reuse ratio (ACRR).

The capacity of cellular systems can be increased by splitting existing cells into smaller cells, thereby reusing frequencies more often in a geographic area but keeping d/r constant. As cells are split, it becomes increasingly difficult to site cells optimally due to the physical network infrastructure and antenna placement requirements [3]. Frequency planning can also become considerably more complex. The lower cell radius limit for many conventional cellular systems was often held to be 1 km [3] although many cellular operators, through necessity, have reduced cell sizes below this level to service their growing customer base.

When cells are less than 1 km in radius they are often described as "microcells." Microcells can provide wireless communications to very large numbers of people at a much higher user density than is possible with macrocells [2], [4]. However, it may become too complex in a microcell system to preallocate channels using FCA [2]. Instead, channels can be globally pooled and allocated at call set up time by the mobile terminal or base station, with the aim of the channel assignment algorithm being the minimization of interference or some other cost function [2]. This is called dynamic channel assignment (DCA).

In FCA systems there is a simple, technology-dependent, relationship between the cluster size C, the CRR d/r, and the signal[1] to interference (s/i) performance of a receiver at a cell boundary in the presence of cochannel interferers [1], [6]–[9]. However, no such simple relationship between cluster size, d/r, and worst case s/i performance exists for DCA systems [9], [10] and the reuse ratio probabilities cannot be predicted using FCA design principles [10].

Researchers have not often examined CRR distributions and probabilities, despite the fact that terminals are randomly located. For example, Linnartz [11] assumed all interfering terminals in an FCA system were located at the nominal reuse distance; Wang and Rappaport [12], [13] assumed terminals were in the "worst case" location in each cell; and Chuang [14] assumed terminals were located at regular fixed points throughout the service area. While this approach may be acceptable for FCA system analysis, it is not clear that it is sufficient for DCA systems.

There are many types of DCA algorithms [2]. The question arises of what happens if, after the algorithm has made its channel choice and it has been successfully assigned to a terminal, that channel is a cochannel or adjacent channel to the channel of an existing user somewhere else in the network. As users move, their original channel assignments will generally not be reconsidered until their radio link conditions, such as s/i, deteriorate to some unacceptable level. But as channels are generally not partitioned in DCA, it is not clear in spatial terms when this will occur.

This paper examines this issue by comparing the CRR and ACRR probabilities in FCA and DCA systems through Monte Carlo simulation and theoretical analysis. The analysis establishes, for the first time, a theoretical lower limit to CRR's

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[1] In this paper, "signal" is used as a synonym for "carrier" as the radio frequency signal, rather than the baseband signal, is always being referred to.

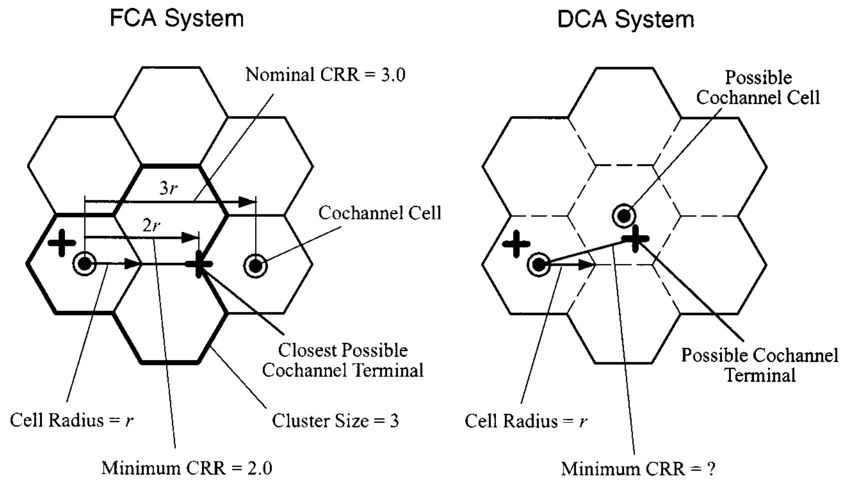

Fig. 1. Cochannel reuse ratio in FCA versus DCA systems.

and ACRR's in DCA cellular systems. This result provides a basis for predicting the closest approach of interferers and the worst case s/i performance for DCA systems, and therefore the quality of the radio coverage offered.

II. Channel Reuse Ratios

A. Channel Reuse in DCA Systems

Channel reuse within DCA and FCA systems is shown in an idealized way in Fig. 1. Each cell is represented as a hexagon, although in practice cells have irregular boundaries. In the FCA system, the available channels are divided into C sets (in Fig. 1 $C = 3$) and the nominal CRR is given by $\sqrt{3C} = 3.0$. As cochannel mobile terminals are confined to the cochannel cell, it can be seen from Fig. 1 that the minimum possible CRR for this FCA system is 2.0.

In DCA systems, however, there is generally no channel partitioning. If the channels are globally pooled, a terminal potentially could use any channel in any cell provided its local radio conditions were satisfactory. As the local radio conditions will depend upon the distribution of terminals and the channel assignments in place, there is no obvious minimum CRR in a DCA system.

B. Monte Carlo Simulation

A computer program has been developed to model arbitrary cellular networks [15]–[19]. The program can be loaded with the technical specifications for existing or proposed cellular technologies and perform a Monte Carlo simulation to estimate system performance parameters such as CRR and ACRR probabilities.

In each simulation, a random sequence of call attempts could be made from mobile terminals randomly placed within the system service area. A mobile terminal's call attempt was deemed to fail if it didn't meet the required signal to noise plus interference ratio $(s/[n+i])$ on both the uplink and downlink. Successful mobile terminals would also drop out if the later success of other terminals leads to an increase in interference and the $s/[n+i]$ falls below threshold. In-cell channel reassignments were performed if the mobile technology specification allowed it.

Note that it is difficult to construct a fair experiment to compare different cellular technologies. Wherever there was not an obvious equivalent basis for comparison (e.g., offered traffic load), the simulation parameters were chosen so that any differences in system performance should be reduced rather than increased.

A system of 21 cells arranged in a regular hexagonal pattern was simulated for four TDMA mobile technologies: GSM (the most widely used digital macrocell system), CT2 (a second-generation digital cordless telephone system), the digital European cordless telephone system (DECT), and the Japanese personal handy phone system (PHS).

For the FCA system (GSM), the cluster size was set to three and the cells were spaced by $\sqrt{3}$ km, giving a target cell radius of 1 km—a small GSM cell. For the DCA systems (CT2, DECT and PHS) the cells were spaced by $100\sqrt{3}$ m, giving a target cell radius of 100 m.

All TDMA systems were assumed to be synchronized, hence there was no intertime-slot interference. Interference between users was purely a result of RF channel spill from other users transmitting on the same time slot.

For each technology simulated, the offered traffic level was set so that approximately 10% of the total number of channels available in each cell would be used simultaneously. Another way of performing the comparison might be to vary the offered traffic levels in each simulation to meet a fixed call loss rate such as 2%. However, this tends to produce less conservative results.

In each simulation, terminals were randomly placed with a uniform area distribution within the 21-cell service area, and each terminal chose the "best" server at call setup time on the basis of received signal strength indication (RSSI). A single exponent distance-dependent path-loss propagation model of the form $P_r \propto P_t d^{-\gamma}$ was used (path-loss exponent $\gamma = 3.0$) so that any difference between FCA and DCA performance would be due to the channel access method rather than the propagation model. Lognormal shadowing and multipath fading were not considered.

TABLE I
COCHANNEL REUSE RATIO SIMULATION RESULTS

Parameter	GSM	CT2	DECT	PHS
Cells	21	21	21	21
Terminals	525	84	252	630
Total Call Loss (%)	1.27	4.28	2.99	0.87
Cochannel Events	2252	6881	11508	7885
Mean CRR	4.17	4.73	4.35	4.54
Standard Deviation CRR	1.32	1.46	1.56	1.51
Maximum CRR	6.98	8.53	8.52	8.51
Minimum CRR	**2.04**	**1.33**	**1.20**	**1.25**

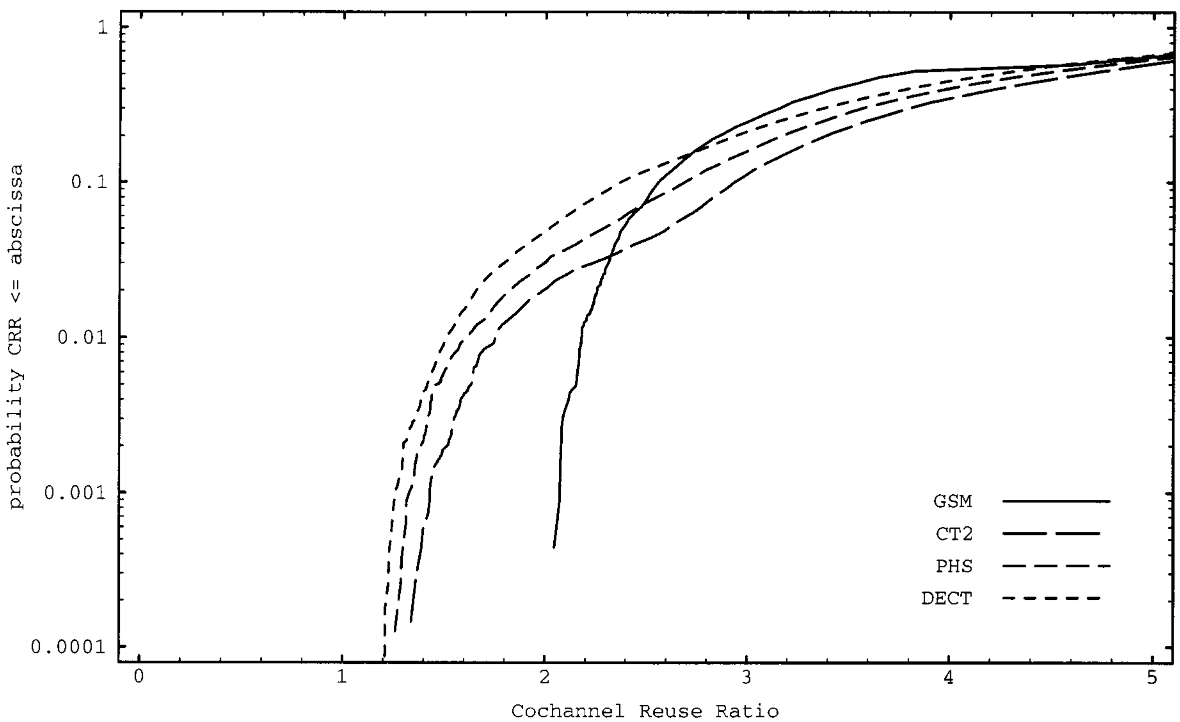

Fig. 2. Cochannel reuse ratio distribution for four mobile technologies.

C. Cochannel Reuse Ratio Distributions

Simulations were performed to estimate the CRR probabilities for GSM, CT2, DECT, and PHS. A total of 10 000 call attempts were made in each simulation, and whenever *any* pair of *successful* terminals were detected to be using the same radio channel and time slot a "cochannel event" was deemed to have occurred and the CRR was calculated. The CRR was calculated as the distance of the interfering terminal from the reference terminal's fixed station divided by the nominal cell radius. The results of these simulations are summarized in Table I.

In the DECT simulation, there were more cochannel events than call attempts. This is because a single terminal can experience more than one cochannel interferer, especially when the total number of terminals exceeds the total number of available channels.

Note that the mean and maximum CRR's are largely determined by the system extent (number of cells) rather than any RF effect. Although the mean CRR was lower for GSM than the DCA systems, the *minimum* CRR was much lower in the DCA systems. Clearly, the actual CRR distribution is critical, as the proportion of terminals successful with small CRR's will affect the radio coverage quality. The cumulative distribution of the CRR for each technology is plotted in Fig. 2 on a log probability scale to highlight the critical area of interest—the CRR for the last 10% of cochannel events.

Examining Fig. 2, it can be seen that the DCA systems exhibit similar CRR distributions, and that a significant proportion of cochannel terminals operated successfully at CRR's smaller than the smallest CRR possible (2.0) for the GSM system modeled. It can be seen that 1.9% of cochannel CT2 terminals, 3.0% of cochannel PHS terminals, and 4.5% of cochannel DECT terminals successfully operated at CRR's of less than 2.0.

Fig. 2 also suggests that RF blocking, or some other mechanism, does eventually place a limit on the minimum CRR in a DCA system, however the lower limit is at a different point for each technology (from 1.20 for DECT to 1.33 for CT2). In Section III, a theoretical analysis is performed to quantify this lower CRR limit.

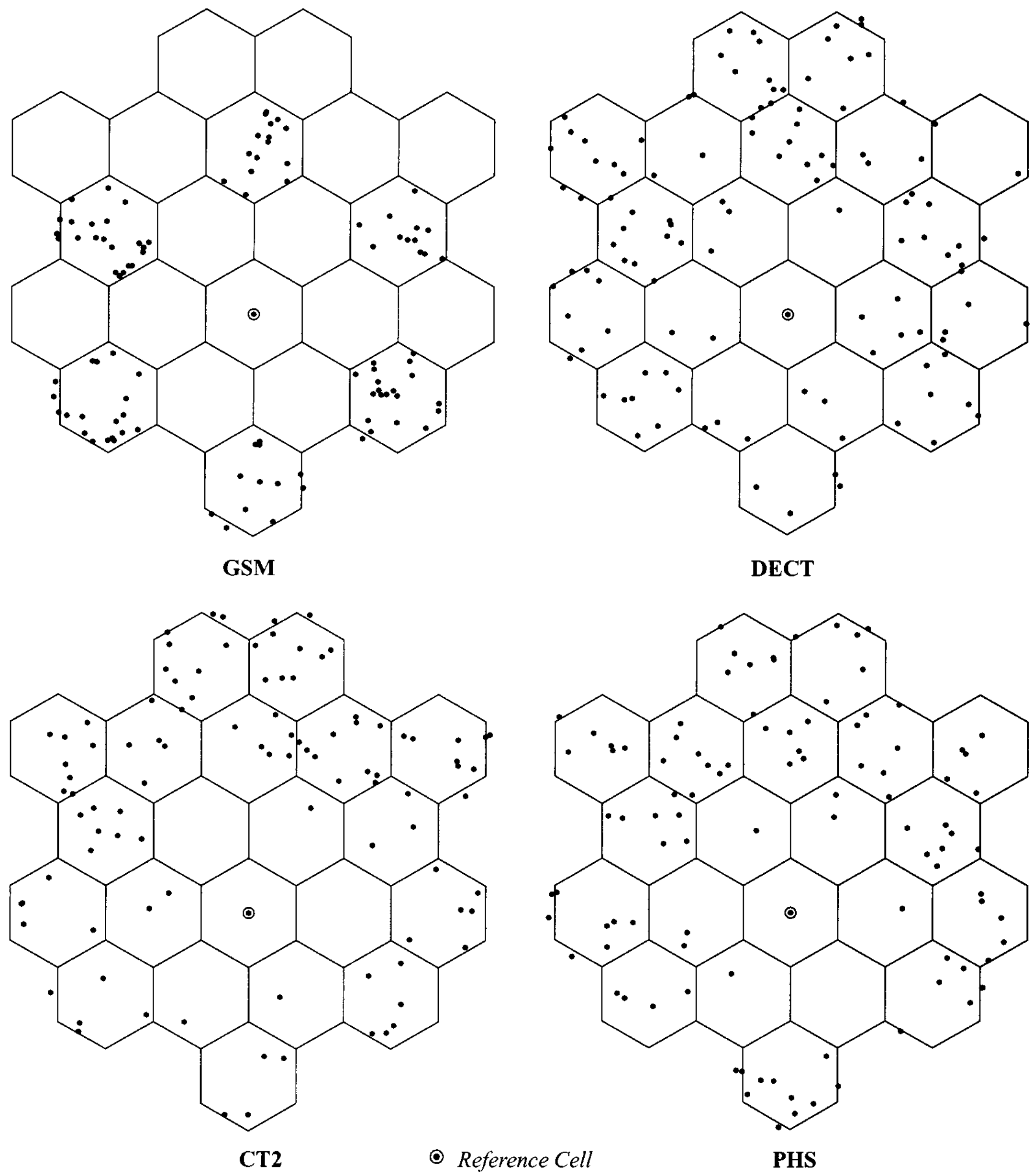

Fig. 3. Location of the first 100 cochannel interferers for GSM and DECT.

The physical nature of the CRR distributions in Fig. 2 can be illustrated by plotting the actual locations of the cochannel interferers with respect to the nominal, idealized, cell boundaries. Fig. 3 shows the location of the first 100 cochannel interferers to users in the central reference cell for the four simulations. The locations of the users in the reference cell are not shown.

In Fig. 3, some of the terminals at the periphery of the cellular service area lie outside of the hexagonal cells. For the purposes of terminal placement in the simulation, the cells were treated as circles rather than hexagons.

The significance of the different CRR probabilities are immediately apparent in Fig. 3. First, the operation of FCA in the GSM system prevents cochannel interferers establishing calls in cells adjacent to the reference cell, and only in six designated first tier cells. In the DCA systems, however, it is clear that cochannel interferers do establish themselves in cells adjacent to the reference cell, and are not restricted in location in first tier cells.

Even though the call loss rates were low for these systems, such close channel reuse compromises coverage quality because close interferers, especially cochannel interferers, severely limit the range of affected terminals [15].

In a DCA system, close cochannel reuse occurs through the statistical fortune of the specific location of the terminals in question (e.g., the reference terminal is close to its fixed station and thus can tolerate high levels of interference). When those terminals move, their limited coverage range could force an in-cell channel reassignment or an intercell handoff.

In TMDA systems, in-cell channel reassignments can either be to a new time slot on the existing radio carrier or to a new time slot on a new carrier, depending upon the technology and reassignment algorithm [20]. In a synchronized TDMA network,

TABLE II
ADJACENT CHANNEL REUSE RATIO SIMULATION RESULTS

Parameter	GSM	CT2	DECT	PHS
Cells	21	21	21	21
Terminals	525	84	252	630
Total Call Loss (%)	1.27	4.28	2.99	0.87
Adjacent Channel Events	5375	18416	17268	19483
Mean ACRR	4.13	4.07	3.89	3.99
Standard Deviation ACRR	1.88	1.73	1.83	1.77
Maximum ACRR	8.53	8.54	8.52	8.50
Minimum ACRR	**0.91**	**0.18**	**0.12**	**0.15**

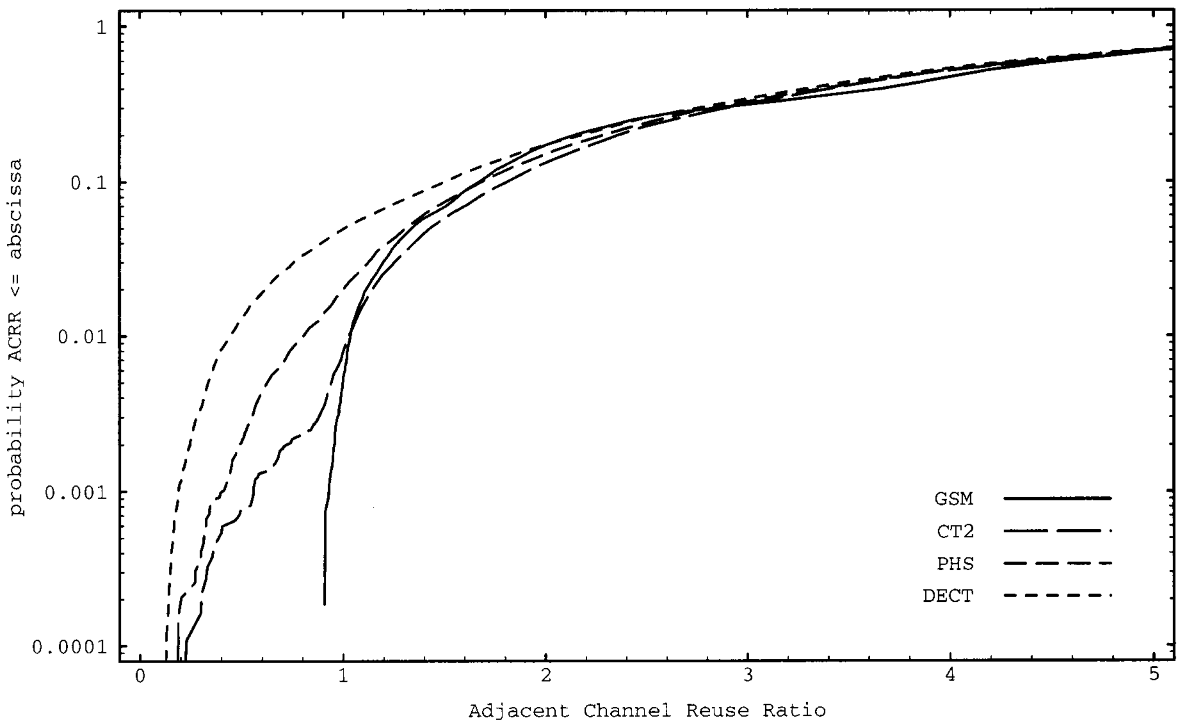

Fig. 4. Adjacent channel reuse ratio distribution for four mobile technologies.

interference is only generated between users on the same radio carrier in different cells if they are also using the same time slot. However, as the number of users and interference levels increase, the probability of unsuccessful in-cell reassignments of both types of intercell handoffs increases [16], [17].

D. Adjacent Channel Reuse Ratio Distributions

In addition to the above CRR data, the *adjacent* channel reuse ratio probabilities for GSM, CT2, DECT, and PHS were collected. "Adjacent" is used here in the sense of the channels *immediately* above or below the reference channel. In this case, each successful terminal may receive interference from one or more other terminals using adjacent channels. The ACRR results are summarized in Table II.

As expected, adjacent channel reuse is possible at much smaller ratios than is possible with cochannels. The DCA systems, however, exhibit very small reuse ratios—as small as 0.12 for DECT, which is significantly smaller than the minimum ACRR achieved with GSM. As with cochannel reuse, adjacent channel reuse is constrained in FCA systems. The typical rule is that adjacent channels cannot be used in the same cell, hence are restricted to being in at least an adjacent cell [1], making the minimum possible ACRR for GSM $\sqrt{3}/2 \simeq 0.87$.

The cumulative distribution of the ACRR for each technology is plotted in Fig. 4 on a log probability scale and Fig. 5 shows the physical location of the first 100 adjacent channel interferers for the GSM and DECT simulations. Examining Fig. 4, it can be seen that although there are differences in the channel reuse ratio probabilities between the four systems, they are less pronounced than those observed for cochannel reuse. DECT and PHS still exhibit ACRR's significantly smaller than GSM at significant probabilties (around 5%). Again, there appears to be a fundamental lower ACRR limit for DCA systems imposed by RF blocking mechanisms.

Fig. 5 plots the location of the first 100 adjacent channel interferers in each simulation to emphasize the physical meaning of the different ACRR distributions. In the GSM network, adjacent channel interferers are not permitted in the reference cell or any of its cochannel cells. In the DECT and PHS networks,

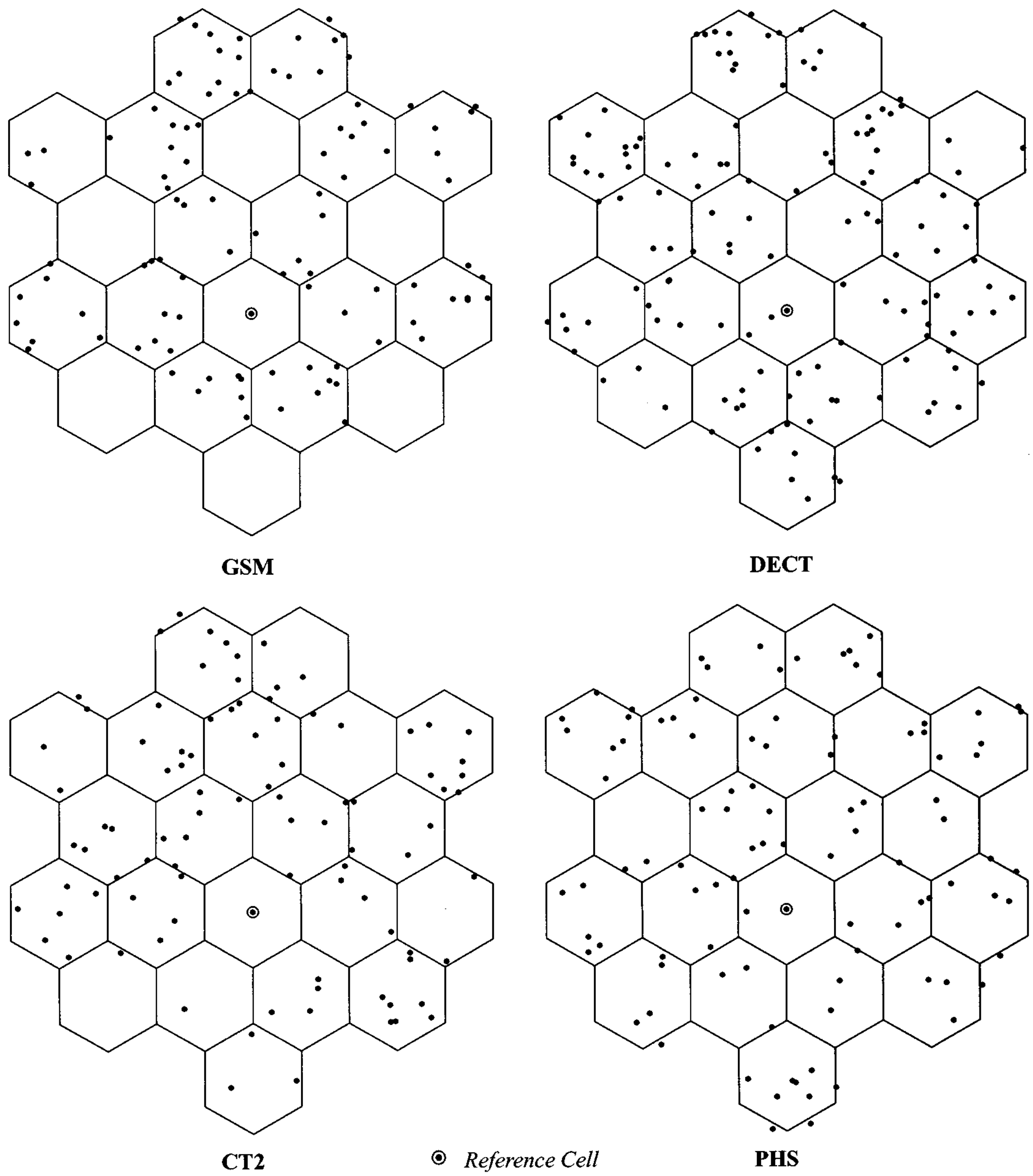

Fig. 5. Location of the first 100 adjacent channel interferers for GSM and DECT.

however, adjacent channel interferers successfully established themselves in the reference cell and all surrounding cells.

The CRR and ACRR results provide fundamental reasons as to why DCA systems exhibit a greater degree of interference domination than conventional cellular systems, e.g., as seen in [19].

III. Channel Reuse Ratio Distribution Analysis

A. Channel Reuse Ratio Model

The factors which influence the channel reuse ratio probabilities include:

- terminal distribution;
- cell layout and service area extent;
- channel assignment algorithm;
- propagation model.

By making simplifying assumptions and following the channel reuse model as shown in Fig. 6, the channel reuse ratio probabilities may be derived analytically. To calculate the reuse probabilities the *a priori* assumption is that a channel reuse event has occurred, hence the terminal M_0 must be in the reference cell (radius r) at some radius ρ from the reference cell site F_0. The interfering terminal M_i is assumed to be within the service area annulus between βr and R and at some radius Π from F_0.

In practice, cell site equipment prevents cochannel reuse within the one cell regardless of whether DCA or FCA is used. Should a cochannel interferer M_i in an adjacent cell move within the boundary of the reference cell, a handover to the reference cell would probably occur and a new channel would be assigned. This has the effect of placing a lower bound on the spatial location of potential cochannel interferers. This is

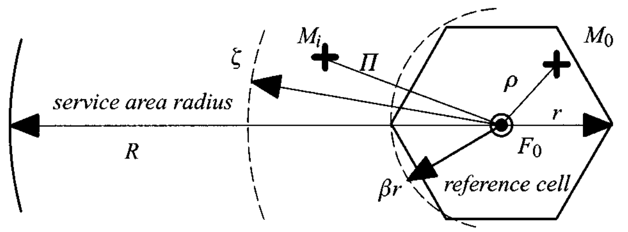

Fig. 6. Channel reuse model.

shown as the inner radius βr in Fig. 6 (β is a dimensionless multiplicative factor).

The signal received at F_0 from M_0 is denoted as s, the interference received at F_0 from M_i as i, and the signal to interference ratio s/i as z, with the random variable (RV) to which it belongs $\mathcal{Z}$. The channel reuse ratio is given by $\Re = \Pi/r$ and the RV to which it belongs is denoted $\mathcal{R}$. For DCA systems the problem is to compute the conditional distribution of the channel reuse ratio

$$F_{\mathcal{R}}(\Re|z \geq Z) = F_{\mathcal{R}}\left(\Re \,\middle|\, \frac{s}{i} \geq Z\right) = \frac{p\left\{\dfrac{\Pi}{r} \leq \Re,\ \dfrac{s}{i} \geq Z\right\}}{p\left\{\dfrac{s}{i} \geq Z\right\}} \tag{1}$$

where

$p\{x\}$ probability of event x occurring;

Z required signal to interference protection ratio for the particular technology.

Note that receiver noise n has been omitted from (1), and hence only a s/i threshold rather than a $s/[n+i]$ threshold is in force. There are two reasons for this. First, cochannel and immediately adjacent channel interferers are much stronger than receiver noise by many orders of magnitude unless the interferer is at a great distance. As the critical area of interest is the probability of achieving *small* reuse ratios, any error resulting from the assumption that $n = 0$ should be necessarily vanishing. Second, if receiver noise n is included an exact closed-form analysis is not possible.

The significance of the radius ζ in Fig. 6 is as follows. There will be some range $\Pi > \zeta$ at which the interferer M_i cannot generate sufficient interference at F_0 to cause M_0's link to fail regardless of M_0's location. Thus, when $\Pi > \zeta$, the signal to interference ratio z will always be greater than or equal to Z and the probabilities $p\{\Pi/r \leq \Re\}$ and $p\{s/i \geq Z\}$ in the numerator of (1) become independent. Conversely, for $\Pi \leq \zeta$ the probabilities in the numerator of (1) are *not* independent and therefore the computation of $p\{\Pi/r \leq \Re,\, s/i \geq Z\}$ becomes more complicated.

The value of ζ may be easily computed assuming a single exponent distance-dependent path-loss propagation model. With this model $s = \kappa P_t \rho^{-\gamma}$ and $i = \alpha\kappa P_t \Pi^{-\gamma}$, where α is the relative strength of the interferer (for a cochannel interferer $\alpha = 1$), κ is an RF constant, P_t is the transmit power and γ is the path-loss exponent. The minimum possible signal power occurs when M_0 is at the periphery of its cell, i.e., $s = \kappa P_t r^{-\gamma}$, hence the minimum signal to interference ratio for M_0 is given by

$$z = \frac{s}{i} = \frac{\kappa P_t r^{-\gamma}}{\alpha\kappa P_t \Pi^{-\gamma}} = \frac{1}{\alpha}\left(\frac{\Pi}{r}\right)^{\gamma} = \frac{1}{\alpha}\left(\frac{r\Re}{r}\right)^{\gamma} = \frac{\Re^{\gamma}}{\alpha}. \tag{2}$$

When $z = Z$

$$\zeta = r\Re = r(\alpha Z)^{1/\gamma} \tag{3}$$

hence the conditional distribution of (1) is piecewise continuous about a reuse ratio $\Re = (\alpha Z)^{1/\gamma}$. It can be shown that when $\Re \leq (\alpha Z)^{1/\gamma}$, (1) can be written as

$$\frac{p\left\{\dfrac{\Pi}{r} \leq \Re,\ \dfrac{s}{i} \geq Z\right\}}{p\left\{\dfrac{s}{i} \geq Z\right\}} = \frac{F_{\mathcal{R}}(\Re) - F_{\mathcal{RZ}}(\Re,\, Z)}{1 - F_{\mathcal{Z}}(Z)}, \qquad \beta \leq \Re \leq (\alpha Z)^{1/\gamma} \tag{4}$$

where $F_{\mathcal{RZ}}(\Re,\, Z)$ is the joint distribution of $\mathcal{R}$ and $\mathcal{Z}$. When $\Re > (\alpha Z)^{1/\gamma}$, (1) needs to be reformulated, and it can be shown that the required distribution becomes

$$F_{\mathcal{R}}\left(\Re \,\middle|\, z \geq Z,\ \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) = \frac{F_{\mathcal{R}}(\Re) - F_{\mathcal{R}}((\alpha Z)^{1/\gamma})}{1 - F_{\mathcal{R}}((\alpha Z)^{1/\gamma})}, \qquad (\alpha Z)^{1/\gamma} < \Re < \frac{R}{r}. \tag{5}$$

Note that (5) is a distribution function in its own right, based upon the assumption that $\Re > (\alpha Z)^{1/\gamma}$. To obtain the *a priori* distribution, (5) must be scaled by $(1-q)$ and shifted by q, where q is the value of (4) at the reuse ratio breakpoint $\Re = (\alpha Z)^{1/\gamma}$.

In FCA systems, the channel reuse ratio distribution is mainly a function of the geometry of the permitted locations for cochannel or adjacent channel interferers. This is because the channel assignment is generally designed such that the required s/i for a mobile terminal is always met, even for worst placed interferers.

For example, in a GSM system with a cluster size of three, the minimum CRR is 2.0 (see Fig. 1). As shown in (3), a mobile terminal in the reference cell will always have its s/i requirement met if $\Re > (\alpha Z)^{1/\gamma}$. For a cochannel GSM interferer, with $Z = 9$ dB and $\gamma = 3.0$, this reuse ratio threshold is $\Re > 2.0$, i.e., the minimum possible reuse ratio with a cluster size $C = 3$.

B. DCA Channel Reuse Ratio Distribution Derivation

For the following derivation, it will be assumed that only *one* interferer exists for any particular channel reuse event, and that it is the dominant interference source (i.e., other interferers and receiver noise n are negligible). For a cochannel interferer $\alpha = 1$, however this parameter will be retained in the derivations for later computation of ACRR's.

Examining (4), the first expression requiring evaluation is $F_{\mathcal{R}}(\Re)$. If it is assumed that M_0 is distributed within the reference cell (radius r) uniformly by area, and that M_i is distributed within the service area annulus between βr and R uniformly by area, it can be shown that

$$F_{\mathcal{R}}(\Re) = \frac{r^2\Re^2 - \beta^2 r^2}{R^2 - \beta^2 r^2}, \qquad \beta \leq \Re \leq \frac{R}{r}. \tag{6}$$

Next, the joint distribution function $F_{\mathcal{R}\mathcal{Z}}(\Re, Z)$ is defined as

$$F_{\mathcal{R}\mathcal{Z}}(\Re, Z) = \int_0^{\Re} \int_{\alpha^{-1}\mathbb{R}^\gamma}^{Z} f_{\mathcal{Z}}(z|\mathcal{R} = \mathbb{R}) f_{\mathcal{R}}(\mathbb{R})\, dz\, d\mathbb{R} \quad (7)$$

where $\mathbb{R} \in \mathcal{R}$ is a dummy variable. The density function $f_{\mathcal{R}}(\Re)$ is simply the derivative, with respect to $\Re$, of the distribution function $F_{\mathcal{R}}(\Re)$ given in (6). Hence

$$f_{\mathcal{R}}(\Re) = \frac{2r^2\Re}{R^2 - \beta^2 r^2}, \qquad \beta \le \Re \le \frac{R}{r}. \quad (8)$$

The conditional density function $f_{\mathcal{Z}}(z|\mathcal{R} = \mathbb{R})$ is the density function of the signal to interference ratio z given a specific reuse ratio and thus a specific amount of interference. Under these conditions the minimum possible value of z is $\alpha^{-1}\mathbb{R}^\gamma$. Given the assumed distributions of M_0 and M_i the density functions of the signal and interference powers can be shown to be

$$f_S(s) = \frac{2}{\gamma r^2}\,(\kappa P_t)^{2/\gamma} s^{-(\gamma+2/\gamma)}, \qquad \kappa P_t r^{-\gamma} \le s < \infty \quad (9)$$

$$f_I(i) = \frac{2}{\gamma(R^2 - \beta^2 r^2)}\,(\alpha\kappa P_t)^{2/\gamma} i^{-(\gamma+2/\gamma)}, \qquad \alpha\kappa P_t R^{-\gamma} \le i < \alpha\kappa P_t(\beta r)^{-\gamma}. \quad (10)$$

With appropriate transformations, the required conditional density function is given by

$$f_{\mathcal{Z}}(z|\mathcal{R} = \mathbb{R}) = \frac{2\alpha\mathbb{R}^2}{\gamma}\,(\alpha z)^{-(\gamma+2/\gamma)}, \qquad \frac{\mathbb{R}^\gamma}{\alpha} \le z < \infty \quad (11)$$

thus the joint distribution function as per (7) may be evaluated to be

$$F_{\mathcal{R}\mathcal{Z}}(\Re, Z) = \frac{r^2}{\alpha^{2/\gamma}(R^2 - \beta^2 r^2)} \cdot \left[\alpha^{2/\gamma}(\Re^2 - \beta^2) + \frac{\Re^4 - \beta^4}{2Z^{2/\gamma}}\right], \qquad \beta \le \Re \le \frac{R}{r}, \quad \frac{\Re^\gamma}{\alpha} \le Z < \infty. \quad (12)$$

The density function of the signal to interference ratio $z = s/i$ is given by [21]

$$f_{\mathcal{Z}}(z) = \int_{-\infty}^{\infty} |i| \cdot f_{SI}(zi, i)\, di = \int_{-\infty}^{\infty} |i| \cdot f_S(zi) \cdot f_I(i)\, di \quad (13)$$

as s and i are independent. It can be shown that $f_{\mathcal{Z}}(z)$ is piecewise continuous about $z = \alpha^{-1}(R/r)^\gamma$, with the expression

$$f_{\mathcal{Z}}(z) = \begin{cases} \dfrac{r^2\alpha^{2/\gamma}}{\gamma z^{(\gamma+2)/\gamma}(R^2 - \beta^2 r^2)}\,[z^{4/\gamma} - \beta^4\alpha^{-(4/\gamma)}], & \dfrac{\beta^\gamma}{\alpha} \le z \le \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma \\ \dfrac{1}{\gamma r^2\alpha^{2/\gamma} z^{(\gamma+2)/\gamma}(R^2 - \beta^2 r^2)}\,[R^4 - \beta^4 r^4], & \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma < z < \infty. \end{cases} \quad (14)$$

The distribution function $F_{\mathcal{Z}}(z)$ can be obtained by integrating (14) with respect to z over the appropriate limits, giving

$$F_{\mathcal{Z}}(z) = \begin{cases} \dfrac{r^2(\beta^2 - \alpha^{2/\gamma}z^{2/\gamma})^2}{2\alpha^{2/\gamma}z^{2/\gamma}(R^2 - \beta^2 r^2)}, & \dfrac{\beta^\gamma}{\alpha} \le z \le \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma \\ 1 - \dfrac{R^4 - \beta^4 r^4}{2r^2\alpha^{2/\gamma}z^{2/\gamma}(R^2 - \beta^2 r^2)}, & \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma < z < \infty. \end{cases} \quad (15)$$

The channel reuse ratio distribution for $\Re \le (\alpha Z)^{1/\gamma}$ computed from (4) is thus

$$F_{\mathcal{R}}(\Re|z \ge Z) = \frac{r^2(\Re^4 - \beta^4)}{2R^2(\alpha Z)^{2/\gamma} - r^2(\alpha Z)^{4/\gamma} - r^2\beta^4}, \qquad \beta \le \Re \le (\alpha Z)^{1/\gamma} \quad (16)$$

and the *a posteriori* channel reuse ratio distribution for $\Re > (\alpha Z)^{1/\gamma}$ computed from (5) is

$$F_{\mathcal{R}}\left(\Re \,\middle|\, z \ge Z,\, \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) = \frac{r^2\Re^2 - r^2(\alpha Z)^{2/\gamma}}{R^2 - r^2(\alpha Z)^{2/\gamma}}, \qquad (\alpha Z)^{1/\gamma} \le \Re \le \frac{R}{\gamma}. \quad (17)$$

As described earlier, (17) is a distribution function in its own right, with a minimum value of zero and a maximum value of unity. As part of the overall reuse ratio distribution, the *a priori* channel reuse ratio distribution for $\Re > (\alpha Z)^{1/\gamma}$ is given by

$$F_{\mathcal{R}}(\Re|z \ge Z) = q + (1-q)F_{\mathcal{R}}\left(\Re \,\middle|\, z \ge Z,\, \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) \quad (18)$$

where

$$q = F_{\mathcal{R}}((\alpha Z)^{1/\gamma}|z \ge Z) = \frac{r^2((\alpha Z)^{4/\gamma} - \beta^4)}{2R^2(\alpha Z)^{2/\gamma} - r^2(\alpha Z)^{4/\gamma} - r^2\beta^4} \quad (19)$$

i.e., the value of the distribution [from (16)] at the reuse ratio breakpoint $\Re = (\alpha Z)^{1/\gamma}$.

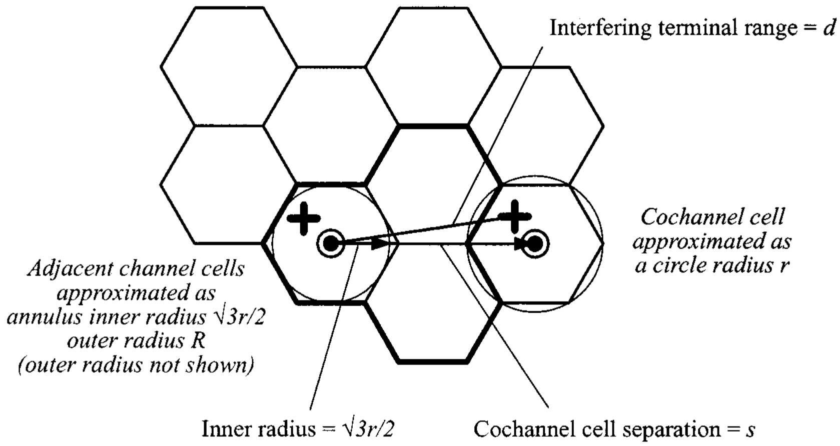

Fig. 7. Channel reuse ratios as a function of cell geometry.

Hence, the complete channel reuse ratio distribution is given by

$$F_{\mathcal{R}}(\Re|z \geq Z) = \begin{cases} \dfrac{r^2(\Re^4 - \beta^4)}{2R^2(\alpha Z)^{2/\gamma} - r^2(\alpha Z)^{4/\gamma} - r^2\beta^4}, \\ \quad \beta \leq \Re \leq (\alpha Z)^{1/\gamma} \\ \dfrac{2r^2\Re^2(\alpha Z)^{2/\gamma} - r^2(\alpha Z)^{4/\gamma} - r^2\beta^4}{2R^2(\alpha Z)^{2/\gamma} - r^2(\alpha Z)^{4/\gamma} - r^2\beta^4}, \\ \quad (\alpha Z)^{1/\gamma} \leq \Re \leq \dfrac{R}{r}. \end{cases} \tag{20}$$

Equation (20) represents the theoretical channel reuse ratio limit for a DCA system with interference protection ratio Z and cells of radius r, operating in a service area of radius R where cochannel terminals are not permitted within a range βr of the reference cell. In practice, channel reuse may not approach this limit, as it was based upon the assumption that a single interferer dominated. Typically, there will be some additional interference which is not negligible [12]. For example, it has been shown that adjacent channel interference can affect the performance of heavily loaded systems [13], [22]–[25].

Note that (20) only applies if $R/r > (\alpha Z)^{1/\gamma}$. If $R/r = (\alpha Z)^{1/\gamma}$ then the first part of (20) covers the complete range of reuse ratios $\Re$ (i.e., $\beta \leq \Re \leq R/r$). If $R/r \leq (\alpha Z)^{1/\gamma}$ a different formulation of the original probability expression, i.e., (4), is required. It can be shown that when $R/r \leq (\alpha Z)^{1/\gamma}$, the channel reuse ratio distribution is given by

$$F_{\mathcal{R}}\left(\Re \middle| z \geq Z, \frac{R}{r} \leq (\alpha Z)^{1/\gamma}\right) = \frac{r^4(\Re^4 - \beta^4)}{R^4 - r^4\beta^4}, \qquad \beta \leq \Re \leq \frac{R}{r}. \tag{21}$$

This is a somewhat counterintuitive result, and is surprising for the fact that it depends upon neither Z nor γ, but solely the geometry of the system ($\beta, r,$ and R). When $R/r \leq (\alpha Z)^{1/\gamma}$, the entire system is inside the radius ζ (as per Fig. 6). The probability of successful channel reuse at a given reuse ratio and the *maximum* possible reuse ratio decrease by the *same proportion*, hence the *shape* of the distribution remains unchanged.

These results does not provide information about *how likely* a channel reuse event is in comparison to nonreuse events. As the system shrinks, there will be fewer cochannel events, but the CDF of the cochannel events that do occur will follow (21). For typical values of Z (less than 20 dB) and systems comprising more than a few cells, $R/r > (\alpha Z)^{1/\gamma}$, rendering this result little more than a curiosity.

C. FCA Channel Reuse Ratio Distribution Derivation

As illustrated in Section III-A, most FCA systems will be designed in such a way that a mobile terminal's s/i requirements will be met even for worst placed interferers. If this is the case, the reuse ratio distribution is independent of the location of the reference terminal and is purely a function of the cell geometry as shown in Fig. 7.

Assuming cochannel terminals are confined to cells of radius r at a range s from the reference cell, the CRR distribution in this case is a simple transformation of a previously obtained distribution for the range d of an interferer [16], [18], and [19] and is given by

$$\begin{aligned} F_{\mathcal{R}}(\Re) = 1 + \frac{1}{\pi r^2} \Bigg\{ & r^2\Re^2 \arccos\left[\frac{r^2(\Re^2-1)+s^2}{2rs\Re}\right] \\ & - r^2 \arccos\left[\frac{r^2(\Re^2-1)-s^2}{2rs}\right] \\ & - rs\sqrt{1-\left(\frac{r^2(\Re^2-1)-s^2}{2rs}\right)^2}\Bigg\}, \\ & \frac{s-r}{r} \leq \Re \leq \frac{s+r}{r}. \end{aligned} \tag{22}$$

Note that (22) applies if, and only if, a link in the reference cell is maintained in the presence of a cochannel interferer at the minimum CRR determined by the FCA algorithm.

The difference between the CRR distributions for DCA systems [(20)] and FCA systems [(22)] is evident and will be examined in Section III-E. The distribution of (20) is a fundamental consequence of channel pooling in DCA. This result proves that the CRR distribution in DCA systems cannot be predicted using conventional cellular engineering techniques.

D. Adjacent Channel Reuse Ratio Distributions

In the case of adjacent channel interferers, it can be seen from Fig. 5 that the FCA algorithm prevents adjacent channel interferers establishing in cochannel cells. This distribution can be approximated by allowing adjacent channel interferers to be located everywhere to the exterior of the reference cell, i.e., an annulus with inner radius $\sqrt{3}r/2$ and outer radius R (Fig. 7). This allows adjacent channel interferers to be located in first tier cochannel cells, but it should not greatly affect the distribution at the smaller reuse ratios.

Assuming such a distribution of adjacent channel interferers, the FCA ACRR distribution can be obtained from simple geometry and is given by

$$F_{\mathcal{R}}(\Re) = \frac{4r^2\Re^2 - 3r^2}{4R^2 - 3r^2}, \qquad \frac{\sqrt{3}}{2} \leq \Re \leq \frac{R}{r}. \tag{23}$$

Again, (23) applies if and only if a link in the reference cell is maintained in the presence of an adjacent channel interferer at the minimum ACRR determined by the FCA algorithm. For the GSM network under consideration, $\alpha = -9$ dB, $Z = 9$ dB and $\gamma = 3$, thus (23) is applicable if $\Re > (\alpha Z)^{1/\gamma}$, i.e., $\Re > 1.0$. The minimum ACRR for this network (see Fig. 7) is $\sqrt{3}/2 \approx 0.87$ so the condition is not quite met, however the error should not be large.

For DCA systems, the ACRR distribution is described by (20) with appropriate values of $\alpha,\ \beta$, and Z substituted. If the DCA system also prevents adjacent channel reuse within a given cell, then the value of β would be set to a value near unity. If the DCA system permitted adjacent channel reuse within a given cell, then the value of β would be set to zero.

E. Theoretical Comparison with Monte Carlo Simulation

The theoretical channel reuse ratio distributions derived in Sections III-B and C can be compared with the Monte Carlo simulation results to assess the degree to which the simplifying assumptions made reduce the accuracy of the results.

First, the DCA CRR and ACRR distributions require the service area radius R to be computed. Examining Fig. 3, it is clear that the service area is not circular and is not centered on the reference cell. As an approximation, the service area comprising hexagonal cells may be replaced with a circle of radius R of equal area, centred on the reference cell as shown in Fig. 8.

It can be shown that T tiers of a C cluster hexagonal cell system consists of $1 + 3T + 3T^2$ cells in total, and each hexagonal cell has an area of $3\sqrt{3}r^2/2$. Hence, a circle of equivalent area requires a radius R of

$$R = \sqrt{\frac{3\sqrt{3}r^2 C(1 + 3T + 3T^2)}{2\pi}}. \tag{24}$$

By substituting the required values in (20) the theoretical CRR and ACRR ratios can be plotted for the DCA systems simulated in Section II-C. First, the systems modeled comprised 1 tier of a (nominal) 3 cell cluster, with each cell 100 m in radius.

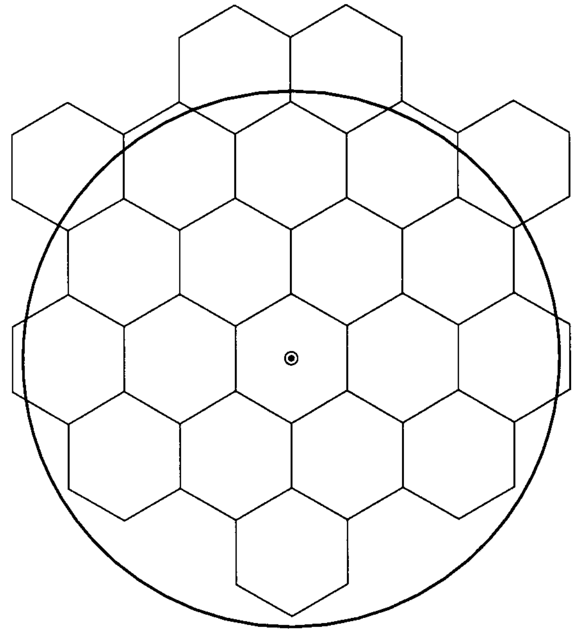

Fig. 8. Approximation of the cellular service area.

Thus, $C = 3$ and $T = 1$ and from (24) the radius of the equivalent service area is $R = 416.7$ m.

Next, the interference protection ratio Z is 14 dB for CT2 [26], 12 dB for PHS [27], and 10 dB for DECT [28]. To evaluate the CRR distribution, $\alpha = 1$ in all cases as cochannel interferers are of equal strength to the signal source. In each simulation the path-loss exponent γ was set to 3.0, thus, for all three systems $Z \leq \alpha^{-1}(R/r)^{\gamma}$ and therefore (20) is applicable.

The parameter β was set to unity to represent the situation where a potential cochannel interferer is always handed over to the reference cell exactly at the reference cell boundary. Depending upon the handover parameters in a live network, the value of β will exhibit statistical fluctuation and may have a mean value different to unity.

For the GSM simulation of Section II-C, the required parameters are $r = 1$ km, $s = 3.0$ km, $Z = 9$ dB [29] and $\gamma = 3.0$. The minimum possible CRR of 2.0 is greater than $Z^{1/\gamma}$ and hence (22) applies. The above CRR distributions were plotted and then superimposed upon the Monte Carlo simulation graph of Fig. 2. The result is shown in Fig. 9.

First, it can be seen in Fig. 9 that the CRR distributions for all DCA technologies are asymptotic to $\Re = 1$ due to the in-cell cochannel reuse constraint. This results in a much more accurate prediction of the probabilities of small CRR's than seen in [30].

Second, the effect of the assumption of only one dominant cochannel interferer can be seen. The Monte Carlo curves for all the DCA technologies lie inside the theoretical curves due to additional, unaccounted for, interference. This has the effect of reducing the probability of successful link establishment at all reuse ratios. This means that the theoretical curves represent the absolute *lower limit* to the CRR in each case, as the minimum amount of cochannel interference is that caused by a single interferer.

Equation (20) therefore enables analytical determination of the minimum possible reuse ratio for a given proportion

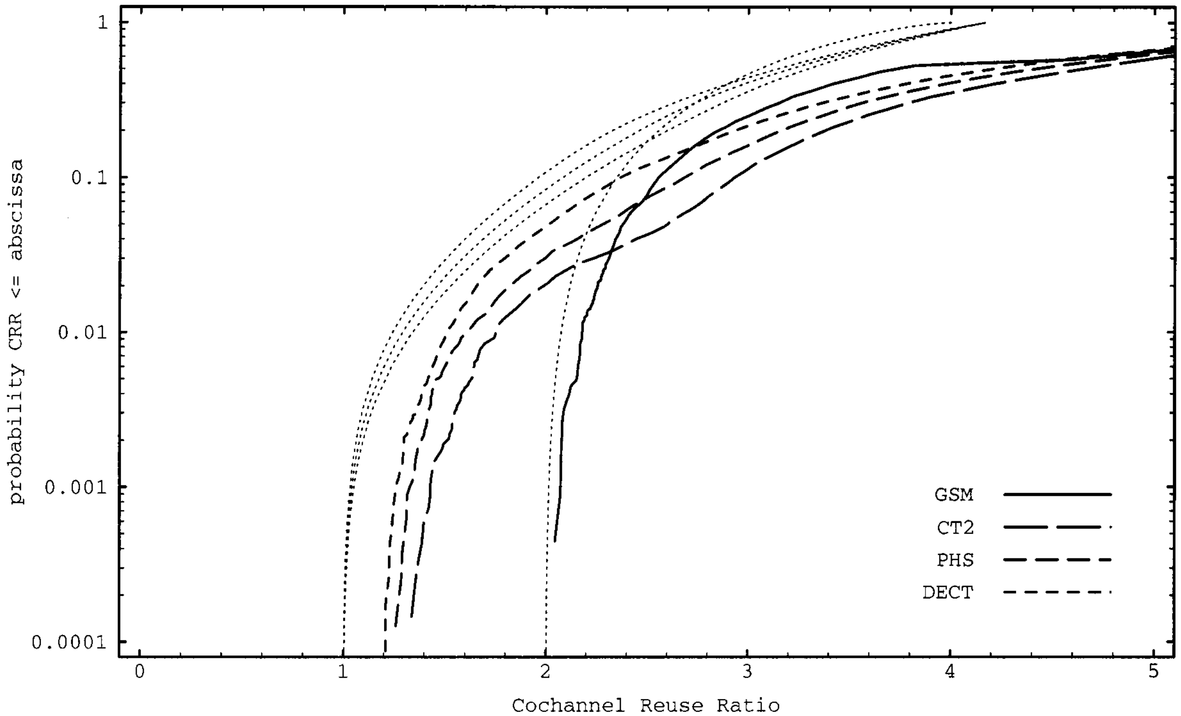

Fig. 9. Theoretical CRR distributions (fine dotted lines) compared with Monte Carlo results (L to R the theoretical curves are for DECT, PHS, CT2, and GSM).

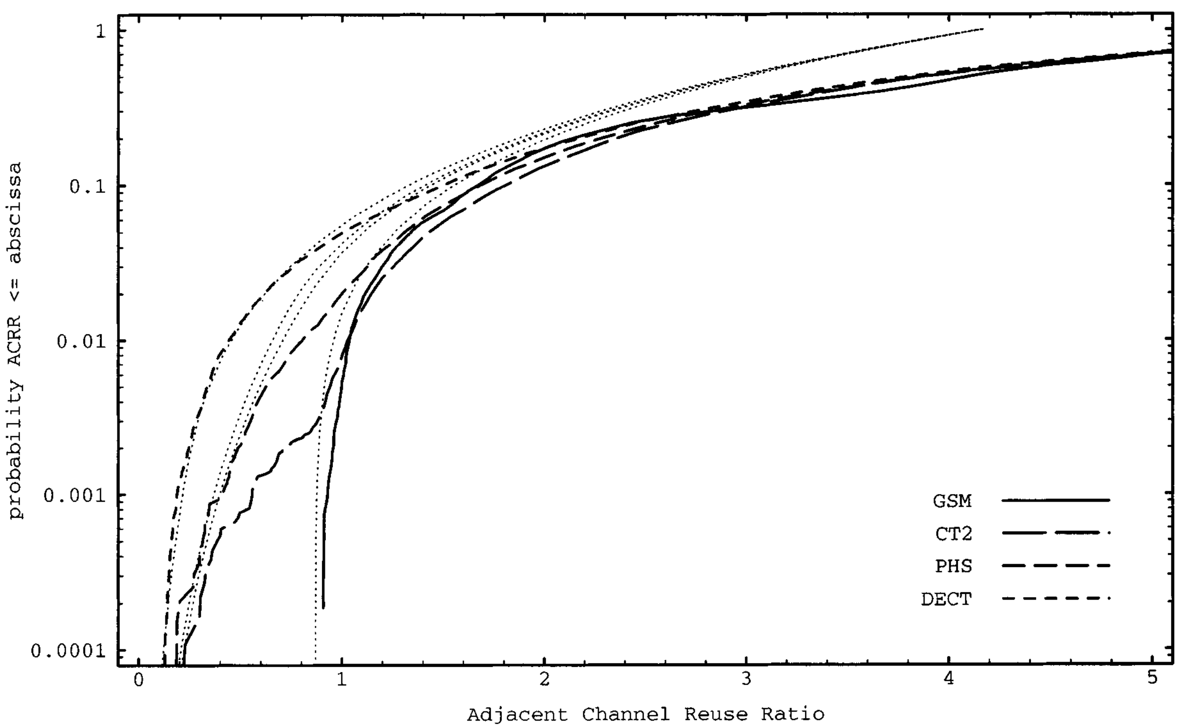

Fig. 10. Theoretical ACRR distributions (fine dotted lines) compared with Monte Carlo results (L to R the theoretical curves are for DECT, PHS, CT2, and GSM).

of cochannel terminals in a DCA cellular system. This is an important result and will assist in the characterization of radio coverage performance in DCA systems. The accuracy of (22) for FCA systems is very good and is a consequence of the fundamental design principles used in FCA systems.

Note that the maximum CRR for the theoretical distributions is R/r (i.e., 4.17 for the current examples) while in the simulations CRR's greater than 8.0 occurred. This is a consequence of the fact that the Monte Carlo simulation sampled *all* cochannel events, not just those in relation to the reference cell. While this affects the accuracy of the theory at large CRR's, it has little effect at small CRR's, which is the area of interest.

The theoretical ACRR distributions can also be plotted for the DCA networks using (20). The relative strength α of adjacent channel interferers varies between the different DCA technologies modeled. The value of α is -16 dB for CT2 [26], -16 dB for PHS [27], and -28 dB for DECT [28]. The parameter β was set to zero to represent the situation where adjacent channel use is permitted in the reference cell. All other parameters are the same as those used for the CRR calculations. For the GSM network, it is assumed that (23) applies for the ACRR distribution. The ACRR distributions were plotted and then superimposed upon the Monte Carlo simulation graph of Fig. 4. The result is shown in Fig. 10.

It can be seen in Fig. 10 that the accuracy of the theoretical results for the ACRR distributions are generally better than for the CRR distributions, particularly for DECT and GSM. The error is largest for CT2 and PHS due to their larger sensitivity to adjacent channel interferers (i.e., larger values of α).

It can be concluded that the differences between the DCA and FCA ACRR distributions are not as pronounced as in the CRR case because there is a smaller correlation between the presence of an adjacent channel interferer and successful link establishment. It is likely that the distributions of further reuse ratios (two channel separation and more) would show even less difference as the effects of the interferers diminish, and the distributions would become dominated by the system geometry.

IV. Conclusion

The CRR and ACRR distributions in DCA cellular systems exhibit very different properties to those found in FCA systems. Computer simulations and mathematical analysis have shown that DCA systems exhibit significantly closer cochannel and adjacent channel frequency reuse than FCA systems for a significant proportion of terminals. This is a fundamental consequence of channel pooling in DCA systems.

Theoretical analysis has provided, for the first time, closed-form expressions for both cochannel and adjacent channel reuse ratio distributions in DCA systems, which in turn establishes a theoretical lower limit to channel reuse ratios in DCA systems. It has been shown that the resultant DCA reuse ratio distributions cannot be obtained using conventional cellular principles.

These results provide a better basis for predicting the probability of the closest approach of cochannel and adjacent channel interferers, enabling better characterization of the worst case s/i performance in DCA cellular systems.

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