

Cochannel and Adjacent Channel Reuse Ratio Distributions in Dynamic Channel Assignment Microcellular Systems

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ABSTRACT: *This paper presents a novel analysis of cochannel and adjacent channel reuse ratio distributions in DCA microcell systems, and compares the results with conventional FCA macrocell systems. Computer simulations show that DCA systems exhibit significantly closer channel reuse than FCA systems for a significant proportion of terminals. Mathematical analysis is used to show that this is a fundamental consequence of the microcell architecture and that the resultant interference distributions cannot be obtained using conventional cellular engineering techniques. A closed form expression for the channel reuse ratio distribution in DCA microcell systems is derived, which in turn establishes a theoretical lower limit to the channel reuse ratio in DCA systems.*

1. INTRODUCTION

The concept of cellular telecommunications, where radio channels are reused at less than horizon distances, was developed by the Bell Telephone Laboratories in the early 1970's [1]. Cellular systems use a large number of radio transmitters, with each transmitter serving a geographic area called a 'cell'. The cells are tessellated to provide radio coverage to an entire city or region.

In conventional cellular systems, the available radio channels are partitioned into C channel sets where C is called the 'cluster size', and each transmitter is allocated the use of one of these channel sets. This is called Fixed Channel Assignment (FCA). Once the system grows beyond C cells, the channel sets are reused in such a way as to not cause excessive interference to existing cells. This is done by keeping the ratio of the distance d (between cells using the same channel set) to the cell radius r at a constant value. The ratio d/r is called the cochannel reuse ratio (CRR).

The capacity of cellular systems can be increased by splitting existing cells into smaller cells, thereby reusing frequencies more often in a geographic area but keeping d/r constant. In practice, however, there is a capacity limit as cells cannot be split indefinitely. The lower cell radius limit for most conventional cell systems (herein referred to as 'macrocells') is in the range of 1 to 1.5 km [2].

Microcellular technologies are being developed to provide wireless communications to very large numbers of people at a much higher user density than is possible with macrocells [3]. Microcell architecture differs from macrocell architecture in three fundamental ways:

- The cells have a much smaller radius
- The mobile terminals radiate much smaller power levels
- The available channels are not partitioned (i.e. all channels are available in every cell)

It is impractical in a microcell system to preallocate channels using FCA. Instead, channels are allocated at call set up time by the mobile terminal or base station, with the aim of the channel assignment algorithm being the minimisation of interference. This is called Dynamic Channel Assignment (DCA).

In FCA systems there is a simple relationship between the cluster size C , the CRR d/r , and the signal to interference (s/i) performance of a receiver at a cell boundary in the presence of cochannel interferers [4–8]. However, no such simple relationship between cluster size, d/r , and worst case s/i performance exists for microcells [7].

This paper compares the cochannel and adjacent channel reuse ratio probabilities in microcell and macrocell systems through Monte Carlo simulation and theoretical analysis. The analysis provides a means of establishing a theoretical limit to cochannel and adjacent channel reuse in DCA microcell systems, and therefore the quality of the radio coverage offered by the system.

2. CHANNEL REUSE RATIOS

2.1 Channel Reuse in DCA Systems

Channel reuse within DCA and FCA systems is shown in an idealised way in Fig. 1. Each cell is represented as a hexagon, although in practice cells have irregular boundaries. In the FCA system, the available channels are divided into C sets (in Fig. 1 $C = 3$) and the nominal CRR is given by $\sqrt{3C} = 3.0$. As cochannel mobile terminals are confined to the cochannel cell, it can be seen in Fig. 1 that the minimum possible CRR for the FCA system is 2.0.

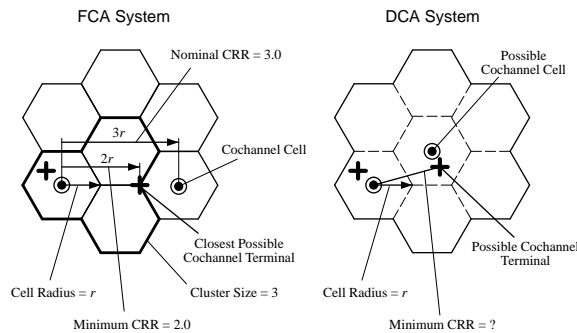


Fig. 1 – Cochannel Reuse Ratio in FCA vs DCA Systems

In FCA systems, the s/i requirement of the particular technology determines the minimum cluster size and the channel assignment pattern. This design principle, however, breaks down in DCA microcells because there is no channel partitioning. Every terminal has the capability of using any channel in any cell and there is not necessarily a guaranteed minimum CRR. The minimum CRR in a microcell systems is a function of pure RF blocking at a receiver (i.e. interference prevents the establishment of a radio link), and the reuse ratio probabilities cannot be predicted using macrocell design principles [9].

Generally researchers do not examine channel reuse ratio *probabilities* despite the fact that terminals are randomly located. For example, Linnartz [10] assumed all interfering terminals in an FCA system were located at the nominal reuse distance; Wang and Rappaport [11,12] assumed terminals were in the ‘worst case’ location in each cell; and Chuang [13] assumed terminals were located at regular fixed points throughout the service area. While this may be acceptable for an FCA system, it is not clear that it is appropriate for DCA systems.

2.2 Monte Carlo Simulation

A computer program has been developed to model arbitrary cellular networks [14–18]. The program can be loaded with the technical specifications for existing or proposed macrocell or microcell technologies, and perform a Monte Carlo simulation to estimate system performance parameters such as cochannel and adjacent channel reuse ratio probabilities.

In each simulation, a random sequence of call attempts can be made from mobile terminals randomly placed within the system service area. A mobile terminal’s call attempt is deemed to fail if it doesn’t meet the required signal to noise plus interference ratio ($s/[n+i]$) on both the uplink and downlink. Initially successful mobile terminals can also drop out if the success of other terminals leads to an increase in interference and the $s/[n+i]$ ratio falls below threshold. In-cell channel reassignments can be performed if the mobile technology specification allows it.

Note that it is difficult to construct a fair experiment to compare different cellular technologies. Wherever there was not an obvious equivalent basis for comparison (e.g. offered traffic load), the simulation parameters were chosen so that any differences in system performance should be reduced rather than increased.

A system of 21 cells arranged in a regular, hexagonal pattern was simulated for four mobile technologies: GSM (the current digital macrocell system used in most of the world); CT2 (a second generation digital cordless telephone system); DECT (the Digital European Cordless Telephone system); and PHS (the Japanese Personal Handy Phone System).

For the FCA system (GSM) the cluster size was set to 3 and the cells were spaced by $\sqrt{3}$ km, giving a target cell radius of 1 km – a very small GSM cell. For the DCA microcell systems (CT2, DECT and PHS) the cells were spaced by $100\sqrt{3}$ m, giving a target cell radius of 100 m. This is towards the lower end of microcell sizes, but not at the limit.

All systems were assumed to be synchronised, hence there was no intertimeslot interference. Interference between users was purely a result of RF channel spill from other users transmitting on the same timeslot.

For each technology simulated, the offered traffic level was set so that approximately 10% of the total number of channels available in that cell would be used simultaneously. Another possible way of performing a fair comparison would be to vary the offered traffic levels in each simulation to meet a fixed call loss rate of, say, 2%. This, however, tends to produce less conservative results.

In each simulation, terminals were randomly placed with a uniform area distribution within the 21 cell service area, and each terminal chose the ‘best’ server at call setup time on the basis of received signal strength indication (RSSI). A single exponent distance dependent path loss propagation model of the form $P_r \propto P_t d^{-\gamma}$ was used, with a path loss exponent $\gamma = 3.0$. Lognormal shadowing and multipath fading were not considered.

2.3 Cochannel Reuse Ratio Distributions

Simulations were performed to estimate the CRR probabilities for GSM, CT2, DECT and PHS. A total of 10000 static call attempts were made in each simulation, and whenever a pair of *successful* terminals were detected to be using the same radio channel and timeslot a ‘cochannel event’ was deemed to have occurred and the CRR was calculated. The CRR was calculated as the distance of the interfering terminal from the wanted link’s fixed station divided by the nominal cell radius. The results of these simulations are summarised in Table I.

TABLE I
COCHANNEL REUSE RATIO SIMULATION RESULTS

Parameter	GSM	CT2	DECT	PHS
Cells	21	21	21	21
Terminals	525	84	252	630
Total Call Loss (%)	1.27	4.28	2.99	0.87
Cochannel Events	2252	6881	11508	7885
Mean CRR	4.17	4.73	4.35	4.54
Std. Dev. CRR	1.32	1.46	1.56	1.51
Maximum CRR	6.98	8.53	8.52	8.51
Minimum CRR	2.04	1.33	1.20	1.25

In the case of DECT there were more cochannel events than call attempts. This is because a single terminal can experience more than one cochannel interferer, especially when the total number of terminals exceeds the total number of available channels.

Note that the mean and maximum CRRs are largely determined by the system extent rather than any RF effect. Although the average CRR was lower for GSM than the microcell systems, the *minimum* CRR was much lower in the microcell systems. Clearly the actual CRR distribution is critical, as the proportion of terminals successful with small CRRs will affect the radio coverage quality. The cumulative distribution of the CRR for each technology is plotted in Fig. 2 on a log probability scale to highlight the critical area of interest – the CRR for the last 10% to 1% of cochannel events.

Examining Fig. 2, it can be seen that the microcell systems exhibit similar CRR distributions, and that a significant proportion of cochannel terminals operated successfully at CRRs smaller than the smallest possible CRR of 2.0 for the GSM system modelled. The simulation results indicated that 2.1% of cochannel CT2 terminals, 3.0% of cochannel PHS terminals, and 4.5% of cochannel DECT terminals successfully operated at CRRs of less than 2.0.

Fig. 2 also suggests that RF blocking, or some other mechanism, does eventually place a limit on the mini-

imum CRR in a DCA system, however the lower limit is at different points for each microcell technology (from 1.21 for DECT to 1.33 for CT2). In Sec. 3 a theoretical analysis is performed to quantify this lower CRR limit.

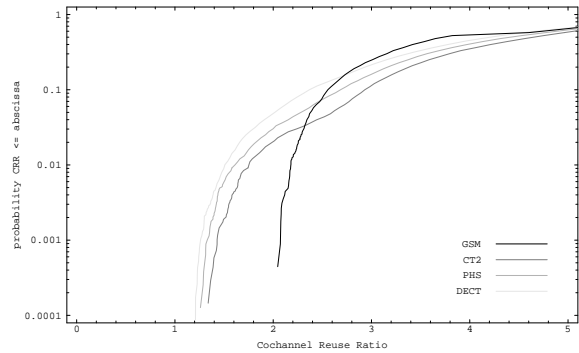


Fig. 2 – CRR distribution for four mobile technologies

The physical nature of the CRR distributions in Fig. 2 can be illustrated by plotting the locations of the cochannel interferers with respect to the nominal, idealised, cell boundaries. Fig. 3 shows the location of the first 100 cochannel interferers to users in the central reference cell for the GSM and DECT simulations. The locations of the users in the reference cell are not shown.

The significance of the different CRR probabilities are immediately apparent in Fig. 3. Firstly, the operation of FCA in the GSM system prevents cochannel interferers establishing calls in cells adjacent to the reference cell, and only in six designated first tier cells. In DECT, however, it is clear that cochannel interferers do establish themselves in cells adjacent to the reference cell, and are not restricted in location in first tier cells.

Even though the call loss rates were low for these systems, such close channel reuse compromises coverage quality because cochannel interference severely limits the terminal range [14,15].

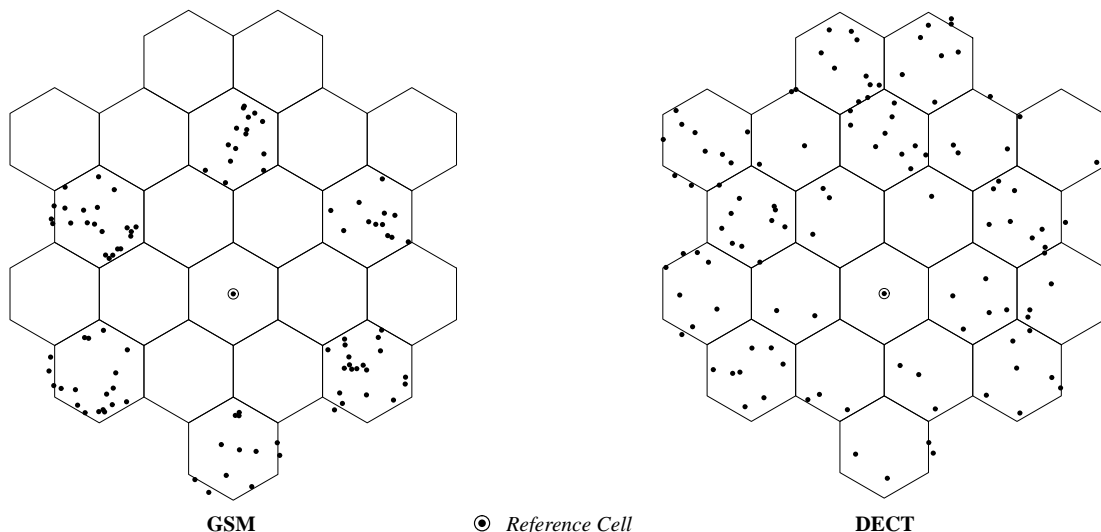


Fig. 3 – Location of the first 100 cochannel interferers for GSM and DECT

In a DCA system, close cochannel reuse occurs through the statistical fortune of the specific location of the terminals in question (e.g. the wanted terminal is close to its fixed station and thus can tolerate high levels of interference). When those terminals move, their limited coverage range could force a handoff or in-cell channel reassignment, and as interference levels increase, the probability increases that such handoffs or reassignments could be unsuccessful [16,17].

2.4 Adjacent Channel Reuse Ratio Distributions

In addition to the above CRR data, the *adjacent* channel reuse ratio (ACRR) probabilities for GSM, CT2, DECT and PHS were also collected. ‘Adjacent’ is used here to describe the channels *immediately* above or below the reference channel. In this case, each successful terminal may receive interference from one or more other terminals using adjacent channels. The ACRR results are summarised in Table II.

TABLE II
ADJACENT CHANNEL REUSE RATIO SIMULATION RESULTS

Parameter	GSM	CT2	DECT	PHS
Cells	21	21	21	21
Terminals	525	84	252	630
Total Call Loss (%)	1.27	4.28	2.99	0.87
Adj. Channel events	5375	18416	17268	19483
Mean ACRR	4.13	4.07	3.89	3.99
Std Dev. ACRR	1.88	1.73	1.83	1.77
Maximum ACRR	8.53	8.54	8.52	8.50
Minimum ACRR	0.91	0.18	0.12	0.15

As expected, adjacent channel reuse is possible at much smaller ratios than is possible with cochannels. However, the microcell systems still exhibit very small reuse ratios – as small as 0.12 for DECT, which is significantly smaller than the minimum ACRR achieved with GSM. As with cochannel reuse, adjacent channel reuse is also constrained in FCA systems. The typical rule is

that adjacent channels cannot be used in the one cell, hence are restricted to being in at least an adjacent cell [8], making the minimum possible ACRR for GSM $\sqrt{3}/2 \approx 0.87$.

The cumulative distribution of the ACRR for each technology is plotted in Fig. 4 on a log probability scale. Examining Fig. 4, it can be seen that although there are differences in the channel reuse ratio probabilities between the four systems, they are less pronounced than that observed for cochannel reuse. DECT and PHS, however, still exhibit ACRRs significantly smaller than GSM at significant probabilities (around 5%). Again, there also appears to be a fundamental lower ACRR limit for DCA systems imposed by RF blocking mechanisms.

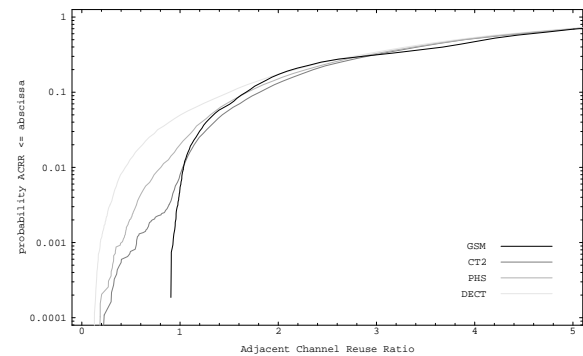


Fig. 4 – ACRR distribution for four mobile technologies

Fig. 5 shows the location of the first 100 adjacent channel interferers for the GSM and DECT simulations, and emphasises the physical meaning of the different ACRR distributions. In the GSM network, adjacent channel interferers are not permitted in the reference cell or any of its cochannel cells. In the DECT network, however, adjacent channel interferers successfully established themselves in the reference cell and all surrounding cells. Again, this leads to microcell systems exhibiting a greater degree of interference domination than macrocell systems.

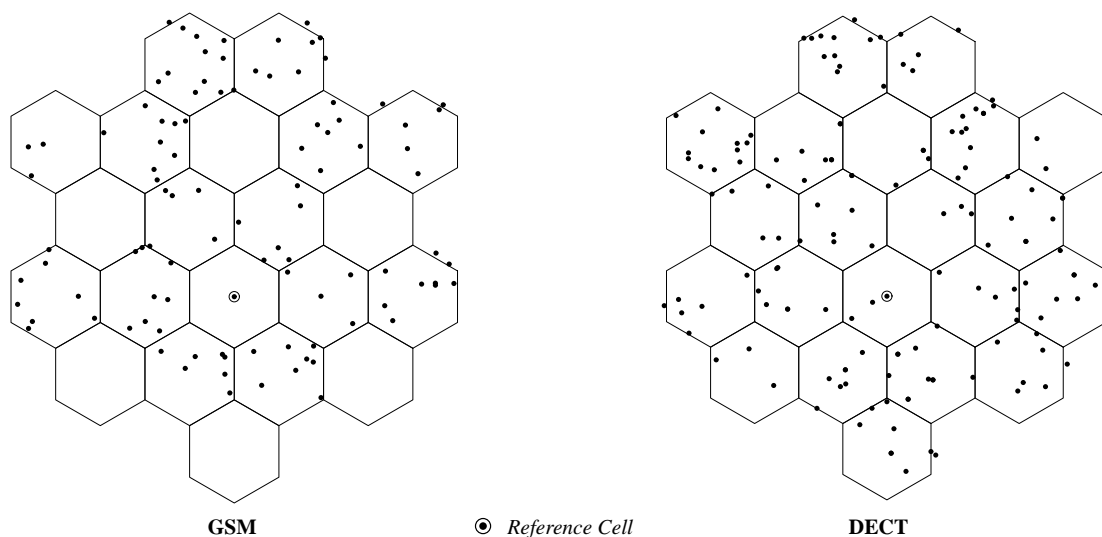


Fig. 5 – Location of the first 100 adjacent channel interferers for GSM and DECT

3. CHANNEL REUSE RATIO DISTRIBUTION ANALYSIS

3.1 Channel Reuse Ratio Model

The factors which influence the channel reuse ratio probabilities include:

- The terminal distribution
- The cell layout and service area extent
- The channel assignment algorithm
- The probability of successful call establishment on a given channel

By making simplifying assumptions and following the channel reuse model as shown in Fig. 6, the channel reuse ratio probabilities may be derived analytically. To calculate the reuse probabilities the *a priori* assumption is that a channel reuse event has occurred, hence the terminal M_0 must be in the reference cell (radius r) at some radius ρ from the reference cell site F_0 . The interfering terminal M_i is assumed to be within the service area of radius R and is at some radius Π from F_0 .

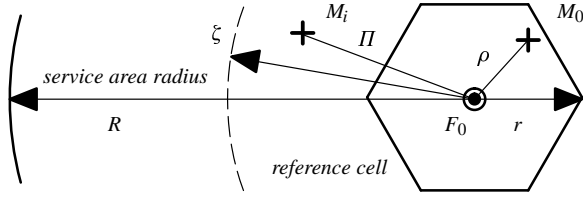


Fig. 6 – Channel reuse model

The signal received at F_0 from M_0 is denoted as s , the interference received at F_0 from M_i as i , and the signal to interference ratio s/i as z , with the random variable (RV) to which it belongs \mathcal{Z} . The channel reuse ratio is given by $\mathfrak{R} = \Pi/r$ and the RV to which it belongs is denoted \mathfrak{R} . For DCA systems the problem is to compute the conditional distribution of the channel reuse ratio, which by definition is:

$$F_{\mathfrak{R}}(\mathfrak{R} \mid z \geq Z) = F_{\mathfrak{R}}\left(\frac{\Pi}{r} \mid \frac{s}{i} \geq Z\right) = \frac{p\left\{\frac{\Pi}{r} \leq \mathfrak{R}, \frac{s}{i} \geq Z\right\}}{p\left\{\frac{s}{i} \geq Z\right\}} \quad (1)$$

where $p\{x\}$ is a probability of event x occurring and Z is the required signal to interference protection ratio for the particular technology.

The significance of the radius ζ in Fig. 6 is as follows. There will be some range $\Pi > \zeta$ at which the interferer M_i cannot generate sufficient interference at F_0 to cause M_0 's link to fail, regardless of M_0 's location. Thus when $\Pi > \zeta$, the signal to interference ratio z will always be greater than or equal to Z and the probabilities $p\{\Pi/r \leq \mathfrak{R}\}$ and $p\{s/i \geq Z\}$ in the numerator of Eq. (1) become independent. Conversely, for $\Pi \leq \zeta$ the

probabilities in the numerator of Eq. (1) are *not* independent and therefore the computation of $p\{\Pi/r \leq \mathfrak{R}, s/i \geq Z\}$ becomes more complicated.

The value of ζ may be easily computed assuming a single-slope distance-dependent path loss propagation model. With this model $s = \kappa P_t r^{-\gamma}$ and $i = \alpha \kappa P_t \Pi^{-\gamma}$, where α is the relative strength of the interferer (for a cochannel interferer $\alpha = 1$), κ is an RF constant, P_t is the transmit power and γ is the path loss exponent. The minimum possible signal power occurs when M_0 is at the periphery of its cell, i.e. $s = \kappa P_t r^{-\gamma}$, hence the minimum signal to interference ratio for M_0 is given by:

$$z = \frac{s}{i} = \frac{\kappa P_t r^{-\gamma}}{\alpha \kappa P_t \Pi^{-\gamma}} = \frac{1}{\alpha} \left(\frac{\Pi}{r}\right)^{\gamma} = \frac{1}{\alpha} \left(\frac{r \mathfrak{R}}{r}\right)^{\gamma} = \frac{\mathfrak{R}^{\gamma}}{\alpha} \quad (2)$$

When $z = Z$,

$$\zeta = r \mathfrak{R} = r(\alpha Z)^{1/\gamma} \quad (3)$$

hence the conditional distribution of Eq.(1) is piecewise continuous about a reuse ratio $\mathfrak{R} = (\alpha Z)^{1/\gamma}$. It can be shown that when $\mathfrak{R} \leq (\alpha Z)^{1/\gamma}$, Eq. (1) can be written as:

$$\frac{p\left\{\frac{\Pi}{r} \leq \mathfrak{R}, \frac{s}{i} \geq Z\right\}}{p\left\{\frac{s}{i} \geq Z\right\}} = \frac{F_{\mathfrak{R}}(\mathfrak{R}) - F_{\mathfrak{R}\mathcal{Z}}(\mathfrak{R}, Z)}{1 - F_{\mathcal{Z}}(Z)} \quad (4)$$

$$0 \leq \mathfrak{R} \leq (\alpha Z)^{1/\gamma}$$

where $F_{\mathfrak{R}\mathcal{Z}}(\mathfrak{R}, Z)$ is the joint distribution of \mathfrak{R} and \mathcal{Z} . When $\mathfrak{R} > (\alpha Z)^{1/\gamma}$, it can be shown that Eq. (1) can be written as:

$$F_{\mathfrak{R}}\left(\mathfrak{R} \mid \frac{\Pi}{r} \geq (\alpha Z)^{1/\gamma}\right) = \frac{F_{\mathfrak{R}}(\mathfrak{R}) - F_{\mathfrak{R}}\left((\alpha Z)^{1/\gamma}\right)}{1 - F_{\mathfrak{R}}\left((\alpha Z)^{1/\gamma}\right)} \quad (5)$$

$$(\alpha Z)^{1/\gamma} < \mathfrak{R} < \frac{R}{r}$$

Note that Eq. (5) is a distribution function in its own right, based upon the assumption that $\mathfrak{R} > (\alpha Z)^{1/\gamma}$. To obtain the *a priori* distribution, Eq. (5) must be scaled by $(1-q)$ and shifted by q , where q is the value of Eq. (4) at the reuse ratio breakpoint $\mathfrak{R} = (\alpha Z)^{1/\gamma}$.

In FCA systems, the channel reuse ratio distribution is mainly a function of the geometry of the permitted locations for cochannel or adjacent channel interferers. This is because the channel assignment is generally designed such that the required s/i for a mobile terminal is always met, even for worst-placed interferers.

For example, in a GSM system with a cluster size of 3, the minimum CRR is 2.0 (see Fig. 1). As shown in Eq. (3), a mobile terminal in the reference cell will always have its s/i requirement met if $\mathfrak{R} > (\alpha Z)^{1/\gamma}$. For a cochannel GSM interferer, with $Z = 9$ dB and $\gamma = 3.0$,

this reuse ratio threshold is $\Re > 2.0$. Hence the condition $\Re > (\alpha Z)^{1/\gamma}$ is always satisfied.

3.2 DCA Channel Reuse Ratio Distribution Derivation

For the following derivation, it will be assumed that only *one* interferer exists for any particular channel reuse event, and that it is the dominant interference source (i.e. other interferers and receiver noise n are negligible). For a cochannel interferer $\alpha = 1$, however this parameter will be retained in the derivations for later computation of ACRRs.

Examining Eq. (4), the first expression requiring evaluation is $F_{\Re}(\Re)$. If it is assumed that M_0 is distributed within the reference cell (radius r) uniformly by area and that M_i is distributed within the service area (radius R) uniformly by area, it can be deduced from first principles that:

$$F_{\Re}(\Re) = \left(\frac{r\Re}{R}\right)^2 \quad (6)$$

Next, the joint distribution function $F_{\Re Z}(\Re, Z)$ is defined as:

$$F_{\Re Z}(\Re, Z) = \int_0^{\Re} \int_{\alpha^{-1}\Re}^Z f_{\Re Z}(z | \Re = \Re) f_{\Re}(\Re) dz d\Re \quad (7)$$

where $\Re \in \Re$ is a dummy variable. The density function $f_{\Re}(\Re)$ is simply the derivative, with respect to \Re , of the distribution function $F_{\Re}(\Re)$ of Eq. (6). Hence:

$$f_{\Re}(\Re) = 2\Re \left(\frac{r}{R}\right)^2 \quad (8)$$

The conditional density function $f_{\Re Z}(z | \Re = \Re)$ is the density function of the signal to interference ratio z given a specific reuse ratio and thus a specific amount of interference. Under these conditions the minimum possible value of z is $\alpha^{-1}\Re^\gamma$. Given the assumed distributions of M_0 and M_i the density functions of the signal and interference powers can be shown to be:

$$f_s(s) = \frac{2}{\gamma r^2} (\kappa P_i)^\frac{2}{\gamma} s^{-\frac{\gamma+2}{\gamma}} \quad \kappa P_i r^{-\gamma} \leq s < \infty \quad (9)$$

$$f_i(i) = \frac{2}{\gamma R^2} (\alpha \kappa P_i)^\frac{2}{\gamma} i^{-\frac{\gamma+2}{\gamma}} \quad \alpha \kappa P_i R^{-\gamma} \leq i < \infty \quad (10)$$

With appropriate transformations, the required conditional density function is given by:

$$f_{\Re Z}(z | \Re = \Re) = \frac{2\alpha\Re^2}{\gamma} (\alpha z)^{-\frac{\gamma+2}{\gamma}} \quad \frac{\Re^\gamma}{\alpha} \leq z < \infty \quad (11)$$

thus the joint distribution function as per Eq. (7) may be evaluated to be:

$$F_{\Re Z}(\Re, Z) = \frac{r^2}{2\alpha^\frac{2}{\gamma} R^2} \left[2\alpha^\frac{2}{\gamma} \Re^2 - \Re^4 Z^{-\frac{2}{\gamma}} \right] \quad (12)$$

$$0 \leq \Re \leq \frac{R}{r}, \quad \frac{\Re^\gamma}{\alpha} \leq Z < \infty$$

The density function of the signal to interference ratio $z = s/i$ is given by:

$$f_{\Re Z}(z) = \int_{-\infty}^{\infty} |i| f_{sI}(zi, i) di = \int_{-\infty}^{\infty} |i| f_s(zi) f_i(i) di \quad (13)$$

as s and i are independent. It can be shown that $f_{\Re Z}(z)$ is piecewise continuous about $z = \alpha^{-1}(R/r)^\gamma$, with the expression:

$$f_{\Re Z}(z) = \begin{cases} \frac{r^2 \alpha^\frac{2}{\gamma}}{\gamma R^2} z^{-\frac{2-\gamma}{\gamma}} & 0 \leq z \leq \frac{1}{\alpha} \left(\frac{R}{r}\right)^\gamma \\ \frac{R^2}{\gamma r^2 \alpha^\frac{2}{\gamma}} z^{-\frac{2+\gamma}{\gamma}} & \frac{1}{\alpha} \left(\frac{R}{r}\right)^\gamma < z < \infty \end{cases} \quad (14)$$

The distribution function $F_{\Re Z}(z)$ can be obtained by integrating Eq. (14) with respect to z over the appropriate limits, giving:

$$F_{\Re Z}(z) = \begin{cases} \frac{r^2 \alpha^\frac{2}{\gamma}}{2R^2} z^\frac{2}{\gamma} & 0 \leq z \leq \frac{1}{\alpha} \left(\frac{R}{r}\right)^\gamma \\ 1 - \frac{R^2}{2r^2 \alpha^\frac{2}{\gamma}} z^{-\frac{2}{\gamma}} & \frac{1}{\alpha} \left(\frac{R}{r}\right)^\gamma < z < \infty \end{cases} \quad (15)$$

The channel reuse ratio distribution for $\Re \leq (\alpha Z)^{1/\gamma}$ computed from Eq. (4) is thus:

$$F_{\Re}(\Re | z \geq Z) = \frac{r^2 \Re^4 (\alpha Z)^{-\frac{2}{\gamma}}}{2R^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}} \quad (16)$$

$$0 \leq \Re \leq (\alpha Z)^{1/\gamma}$$

and the *a posteriori* channel reuse ratio distribution for $\Re > (\alpha Z)^{1/\gamma}$ computed from Eq. (5) is:

$$F_{\Re}(\Re | \frac{\Re}{r} \geq (\alpha Z)^{\frac{1}{\gamma}}) = \frac{r^2 \Re^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}}{R^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}} \quad (17)$$

$$(\alpha Z)^{1/\gamma} \leq \Re \leq \frac{R}{r}$$

As described earlier, Eq. (17) is a distribution function in its own right, with a minimum value of zero and a maximum value of unity. As part of the overall reuse ratio distribution, the *a priori* channel reuse ratio distribution for $\Re > (\alpha Z)^{1/\gamma}$ is given by:

$$F_{\Re}(\Re | z \geq Z) = q + (1 - q)F_{\Re}\left(\Re \mid \frac{\Pi}{r} \geq (\alpha Z)^{1/\gamma}\right) \quad (18)$$

where

$$q = F_{\Re}\left((\alpha Z)^{1/\gamma} \mid z \geq Z\right) = \frac{r^2(\alpha Z)^{\frac{2}{\gamma}}}{2R^2 - r^2(\alpha Z)^{\frac{2}{\gamma}}} \quad (19)$$

i.e. the value of the distribution (from Eq. (16)) at the reuse ratio breakpoint $\Re = (\alpha Z)^{1/\gamma}$.

Hence the complete channel reuse ratio distribution is given by:

$$F_{\Re}(\Re \mid z \geq Z) = \begin{cases} \frac{r^2 \Re^4 (\alpha Z)^{-\frac{2}{\gamma}}}{2R^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}} & 0 \leq \Re \leq (\alpha Z)^{1/\gamma} \\ \frac{2r^2 \Re^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}}{2R^2 - r^2 (\alpha Z)^{\frac{2}{\gamma}}} & (\alpha Z)^{1/\gamma} \leq \Re \leq \frac{R}{r} \end{cases} \quad (20)$$

Eq. (20) represents the theoretical channel reuse ratio limit for a DCA system with interference protection ratio Z and cells of radius r operating in a service area of radius R . In practice, channel reuse may not approach this limit, as it was based upon the assumption that a single interferer dominated. Typically, there will be some additional interference which is not negligible [11]. For example, it has been shown that adjacent channel interference can affect the performance of heavily loaded systems [12,19–22].

Note that Eq. (20) only applies if $R/r > (\alpha Z)^{1/\gamma}$. If $R/r = (\alpha Z)^{1/\gamma}$ then the first part of Eq. (20) covers the complete range of reuse ratios \Re (i.e. $0 \leq \Re \leq R/r$). If $R/r \leq (\alpha Z)^{1/\gamma}$ a different formulation of the original probability expression, i.e. Eq. (4), is required. However, for typical values of Z (less than 20 dB) and systems comprising more than a few cells, $R/r > (\alpha Z)^{1/\gamma}$.

3.3 FCA Channel Reuse Ratio Distribution Derivation

As illustrated in Sec. 3.1, most FCA systems will be designed in such a way that a mobile terminal's s/i requirements will be met even for worst placed interferers. If this is the case, the reuse ratio distribution is independent of the location of the reference terminal and is

purely a function of the cell geometry as shown in Fig. 7.

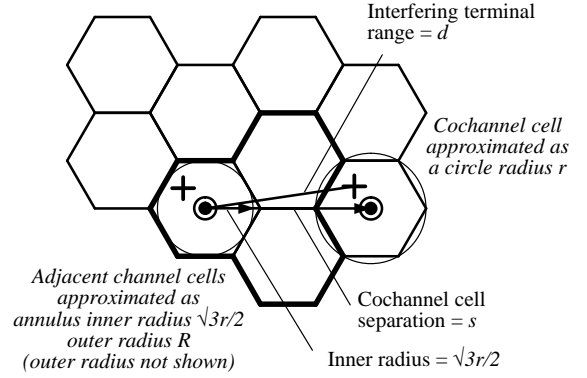


Fig. 7 – Channel reuse ratios as a function of cell geometry

Assuming cochannel terminals are confined to cells of radius r at a range s from the reference cell, the CRR distribution in this case is a simple transformation of a previously obtained distribution for the range d of an interferer [16,18] and is given by:

$$F_{\Re}(\Re) = 1 - \frac{1}{\pi r^2} \left\{ r^2 \cos\left[\frac{r^2(\Re^2 - 1) - s^2}{2rs}\right] - r^2 \Re^2 \cos\left[\frac{r^2(\Re^2 - 1) + s^2}{2rs\Re}\right] + rs \sqrt{1 - \left(\frac{r^2(\Re^2 - 1) - s^2}{2rs}\right)^2} \right\} \quad (21)$$

$$\frac{s-r}{r} \leq \Re \leq \frac{s+r}{r}$$

Note that Eq. (21) applies if and only if a link in the reference cell is maintained in the presence of a cochannel interferer at the minimum CRR determined by the FCA algorithm.

The difference between the CRR distributions for DCA systems (Eq. (20)) and FCA systems (Eq. (21)) is evident. The distribution of Eq. (20) is a fundamental consequence of the lack of channel partitioning in DCA. This result proves that the CRR distribution in DCA systems cannot be predicted using macrocell engineering techniques.

In the case of adjacent channel interferers, it can be seen from Fig. 5 that the FCA algorithm prevents adjacent channel interferers establishing in cochannel cells. This distribution can be approximated by allowing adjacent channel interferers to be located everywhere to the exterior of the reference cell, i.e. an annulus with inner radius $\sqrt{3}r/2$ and outer radius R (see Fig. 7). This allows adjacent channel interferers to be located in first tier cochannel cells, but this should not greatly affect the distribution at the smaller reuse ratios.

Assuming such a distribution of adjacent channel interferers, the ACRR distribution can be obtained from simple geometry and is given by:

$$f_{\mathfrak{R}}(\mathfrak{R}) = \frac{4r^2\mathfrak{R}^2 - 3r^2}{4R^2 - 3r^2} \quad \frac{\sqrt{3}}{2} \leq \mathfrak{R} \leq \frac{R}{r} \quad (22)$$

Again, Eq. (22) applies if and only if a link in the reference cell is maintained in the presence of an adjacent channel interferer at the minimum ACRR. For example, assume that in the GSM network under consideration, $\alpha = -9$ dB, $Z = 9$ dB and $\gamma = 3$. Under these conditions, Eq. (22) is applicable if $\mathfrak{R} > (\alpha Z)^{1/\gamma}$, i.e. $\mathfrak{R} > 1.0$. The minimum ACRR in an FCA system with $C = 3$ is $\sqrt{3}/2 \approx 0.87$ (see Fig. 7). Hence the condition $\mathfrak{R} > 1.0$ is not quite met, however the resulting error in the theoretical ACRR distribution should be small.

3.4 Theoretical Comparison with Monte Carlo Simulation

The theoretical channel reuse ratio distributions derived in Secs. 3.2 and 3.3 can be compared with the Monte Carlo simulation results to assess the degree to which the simplifying assumptions made reduce the accuracy of the results.

Firstly, the DCA CRR and ACRR distributions require the service area radius R to be computed. Examining Fig. 3 it is clear that the service area is not circular and is not centred on the reference cell. As an approximation, the service area comprising hexagonal cells may be replaced with a circle of radius R of equal area centred on the reference cell as shown in Fig. 8.

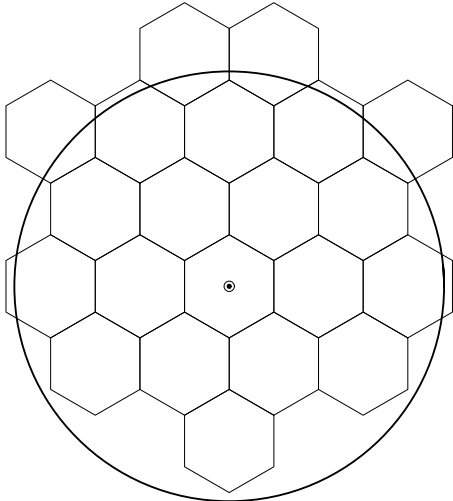


Fig. 8 – Approximation of the cellular service area

It can be shown that T tiers of a C cluster hexagonal cell system consists of $1+3T+3T^2$ cells in total, and each hexagonal cell has an area of $3\sqrt{3}r^2/2$. Hence a circle of equivalent area requires a radius R of:

$$R = \sqrt{\frac{3\sqrt{3}r^2C(1+3T+3T^2)}{2\pi}} \quad (23)$$

By substituting the required values in Eq. (20) the theoretical CRR and ACRR ratios can be plotted for the DCA systems simulated in Sec. 3.3. Firstly, the systems modelled comprised 1 tier of a (nominal) 3 cell cluster, with each cell 100 m in radius. Thus $C = 3$ and $T = 1$ and from Eq. (23) the radius of the equivalent service area is $R = 416.7$ m.

Next, the interference protection ratio Z is 14 dB for CT2 [23], 12 dB for PHS [24], and 10 dB for DECT [25]. To evaluate the CRR distribution $\alpha = 1$ in all cases as cochannel interferers are of equal strength to the signal source. In each simulation the path loss exponent γ was set to 3.0, thus for all three systems $Z \leq \alpha^{-1}(R/r)^\gamma$ and therefore Eq. (20) is applicable.

For the GSM simulation of Sec. 3.3, the required parameters are $r = 1$ km, $s = 3.0$ km, $Z = 9$ dB [26] and $\gamma = 3.0$. The minimum possible CRR of 2.0 is greater than $Z^{1/\gamma}$ and hence Eq. (21) applies. The above CRR distributions were plotted and then superimposed upon the Monte Carlo simulation graph of Fig. 2. The result is shown in Fig. 9 below.

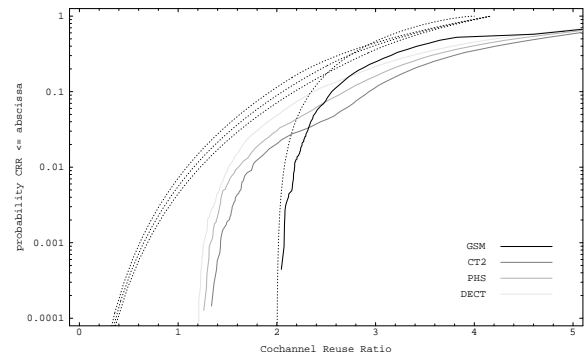


Fig. 9 – Theoretical CRR distributions (dotted lines) compared with Monte Carlo results (L to R the theoretical curves are for DECT, PHS, CT2 and GSM).

The effect of the assumption of only one dominant cochannel interferer can be seen. The Monte Carlo curves for all the microcell technologies are inside the theoretical curves due to additional, unaccounted for, interference. This has the effect of reducing the probability of successful link establishment at all reuse ratios. This means, however, the theoretical curves represent the absolute *lower limit* to the CRR in each case as the minimum amount of cochannel interference is that caused by a single cochannel interferer.

Eq. (20) therefore enables analytical determination of the minimum possible reuse ratio for a given proportion of cochannel terminals in a DCA microcell system. This is an important result and will assist in the characterisation of radio coverage performance in microcell systems. The accuracy of Eq. (21) for FCA systems is very good and is a consequence of the fundamental design principles used in FCA systems.

Note that the maximum CRR for the theoretical distributions is R/r (i.e. 4.17 for the current examples) while in the simulations CRRs of over 8.0 occurred. This is a consequence of the fact that the Monte Carlo simulation sampled *all* cochannel events, not just those

in relation to the reference cell. While this affects the accuracy of the theory at large CRRs, it has little effect at small CRRs, which is the area of interest.

The theoretical ACRR distributions can also be plotted for the microcell networks using Eq. (20). The relative strength α of adjacent channel interferers varies between the different microcell technologies modelled, otherwise the parameters are the same as for the CRR calculations. The value of α is -16 dB for CT2 [23], -16 dB for PHS [24], and -28 dB for DECT [25]. For the GSM network, it is assumed Eq. (22) applies for the ACRR distribution. The ACRR distributions were plotted and then superimposed upon the Monte Carlo simulation graph of Fig. 4. The result is shown in Fig. 10 below.

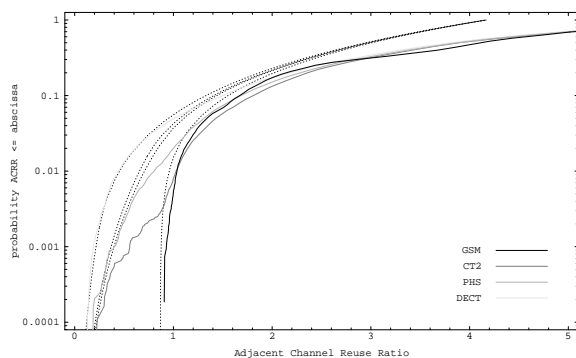


Fig. 10 – Theoretical ACRR distributions (dotted lines) compared with Monte Carlo results (L to R the theoretical curves are for DECT, PHS, CT2 and GSM).

The accuracy of the theoretical results for the ACRR distributions are generally better than for the CRR distributions, particularly for DECT and GSM. The error is largest for CT2 and PHS due to their larger sensitivity to adjacent channel interferers (i.e. larger values of α).

It can be concluded that the differences between the DCA and FCA ACRR distributions are not as pronounced as in the CRR case because there is a smaller correlation between the presence of an adjacent channel interferer and successful link establishment. It is likely that the distributions of further reuse ratios (two channel separation and more) would show even less difference as the effects of the interferers diminish, and the distributions would become dominated by the system geometry.

4. CONCLUSION

Cochannel and adjacent channel reuse ratio distributions in DCA microcell systems exhibit very different properties to those found in FCA macrocell systems. Computer simulations and mathematical analysis has shown that DCA systems exhibit significantly closer cochannel and adjacent channel frequency reuse than FCA systems for a significant proportion of terminals. This is a fundamental consequence of the lack of channel partitioning in DCA

Theoretical analysis has provided closed form expressions for both cochannel and adjacent channel reuse ratio distributions in DCA systems, which in turn establishes a theoretical lower limit to the channel reuse ratio in DCA systems. It has been shown that the resultant reuse ratio distributions cannot be obtained using conventional macrocell engineering principles.

These results enable the probability of the closest approach of interferers of arbitrary strength to be predicted for microcell systems. This may enable characterisation of the worst case s/i performance in DCA microcell systems, and enable better modelling of the radio coverage performance of DCA microcell systems.

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