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Interference Distributions in Microcell Ensembles

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Abstract: **An interference model developed for arbitrary microcellular networks, based upon a parameter called the ‘Interference to Noise Ratio’ or INR, is used to derive interference and cell radius statistics for mobile stations in a microcellular network.**

The theoretical INR and cell radius statistics for simple interference environments show good agreement with numerical Monte Carlo simulations. For more complex environments, it is hypothesised that the Central Limit Theorem could be applied to approximate the behaviour of an interferer ensemble, enabling the cell radius statistics for a network to be expressed in terms of the system design parameters. Simulations of microcell ensembles that support this hypothesis are presented.

I. INTRODUCTION

Microcellular technologies are being developed with a view to satisfy the vision of a personal communicator in every pocket. The aims of a future Personal Communications Service (PCS) provided by a ubiquitous microcell network include [1]:

- Low cochannel interference (< 1% of users)
- Closer frequency reuse
- Large percentage coverage (> 99%)

The wide scale deployment of a high grade wireless telephone system will require engineering tools and techniques that allow rapid and accurate system design, and the fundamental problem that needs to be addressed is of modelling the end result of multiple users propagating in a congested area [2]–[3].

Consideration of service quality, such as quality of cell coverage, is an imperative because as the user base increases people will no longer accept poor call quality simply because the service is ‘mobile’ [4] but will demand a similar grade of service to that experienced in wireline services [3].

II. SYSTEM DESIGN ISSUES

There does not yet appear to be a systematic design methodology for engineering a ubiquitous microcell network to a target Grade of Service or Quality of Service [2]–[7], and the applicability of conventional cellular design techniques to the microcellular case is questionable.

Firstly, unlike in large cell systems, there is no simple relationship between the cluster size C and the worst case S/I at a receiver in microcells [8]. Interference control techniques such as Dynamic Channel Allocation (DCA) can still fail under heavy traffic loads [9], leading to excessive interference between users and irregularly shaped microcells [5].

Secondly, it has been shown that adjacent channel interference (ACI) can affect the performance of heavily loaded large cell systems [10]–[12]. With irregular microcell locations and unknown mobile positions, the effects of ACI and further off–channel interference could be even worse [13].

Thirdly, the spatial traffic variability and close spacing of base stations in microcellular systems (especially in multioperator environments) have a significant impact upon the percentage of service area that has a circuit quality better than some specified value [1],[5]–[7],[14].

These factors may make it very difficult to engineer a microcell system so that reliable, contiguous radio coverage is achieved. It has been suggested that ubiquitous coverage may be nearly impossible to achieve [15].

An interference model incorporating the cumulative interference effects of all users in an arbitrary microcellular network [16]–[19] provides a means of analysing microcell coverage quality. This model considers both thermal noise and propagated interference because the transmission quality is strongly dependent upon these two factors [20].

III. GENERAL MICROCELL INTERFERENCE MODEL

Consider a microcell network model [18] consisting of a mobile terminal M_{00} attempting to establish a link with a fixed station F_0 in the presence of n additional fixed stations F_i $\{1 \le i \le n\}$ where each fixed station communicates with a number of additional mobile terminals M_{ij} $\{1 \le i \le n, 1 \le j \le m_i\}$ where m_i is the total number of mobile stations communicating with fixed station F_i as shown in figure 1.

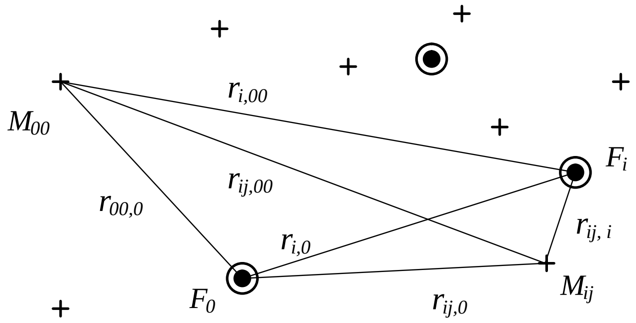

Figure 1 – Arbitrary microcell network

Each mobile transmits at a power P_{Mij} and uses channel C_{Uij} on the uplink (M_{ij} to F_i) and C_{Dij} on the downlink (F_i to M_{ij}). These channels may be physical RF channels, TDMA timeslots, or CDMA sequences [18]. Fixed stations transmit at a power P_{Fij} to mobile terminal M_{ij}. The notation used for distances r between transmitters and receivers is as shown in figure 1.

Noise in the receivers is considered as a constant interference source with a mean power of N. At a receiver, a link will be suc-

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cessful if the signal to noise plus interference ratio $S/[N+I]$ is greater than or equal to the system protection ratio Z, otherwise an outage occurs. For analytical tractability a simple distance–dependent propagation model $P_{rx} = \kappa P_{tx} d^{-\gamma}$ was used in [16]–[19], where the exponent γ is the path loss exponent and κ is an RF factor.

All channel spills (i.e. cochannel, adjacent channel, and all further off–channel spill) are considered between every transmitter and receiver in the model. It has been shown [16],[18] that the uplink (M_{00} to F_0) outage contour satisfies the expression:

$$r^{\gamma}_{00,0_F} = \psi_{M_{00}} \left[\frac{1}{\eta_{F_{00}} + 1} \right] \tag{1}$$

and the downlink (F_0 to M_{00}) outage contour satisfies the expression:

$$r^{\gamma}_{00,0_M} = \psi_{F_{00}} \left[\frac{1}{\eta_{M_{00}} + 1} \right] \tag{2}$$

where $\psi_{M00} = \kappa P_{M00}/ZN$ for the uplink and $\psi_{F00} = \kappa P_{F00}/ZN$ for the downlink. The dimensionless parameter η is the Interference to Noise Ratio or 'INR' at the receiver in question, and for the uplink and downlink respectively, it is given by [18]:

$$\eta_{F_{00}} = \frac{1}{N} \left\{ \sum_{i=1}^{n} \left[\sum_{j=1}^{m_i} \left[\kappa_{ij} \; P_{F_{ij} F_{00}} \right] r^{-\gamma}_{i,0} \right] + \sum_{i=0}^{n} \left[\sum_{j=1}^{m_i} \left[\kappa_{ij} \; P_{M_{ij} F_{00}} \; r^{-\gamma}_{ij,0} \right] \right] \right\} \tag{3}$$

$$\eta_{M_{00}} = \frac{1}{N} \left\{ \sum_{i=0}^{n} \left[\sum_{j=1}^{m_i} \left[\kappa_{ij} \; P_{F_{ij} M_{00}} \right] r^{-\gamma}_{i,00} \right] + \sum_{i=0}^{n} \left[\sum_{j=1}^{m_i} \left[\kappa_{ij} \; P_{M_{ij} M_{00}} \; r^{-\gamma}_{ij,00} \right] \right] \right\} \tag{4}$$

The INR is the ratio of total received interference power to receiver noise at the receiver, and provides a seamless means of dealing with all interference conditions from purely noise limited ($\eta = 0$) to purely interference limited ($\eta \rightarrow \infty$). The P_{FF} terms in equations (3) and (4) refer to fixed–to–fixed station channel spill, P_{MF} mobile–to–fixed channel spill, etc.

Equation (1) describes a circle centred on F_0 and equation (2) describes a higher plane curve that can usually only be computed numerically. For a duplex link to be successful, both ends must satisfy their respective outage conditions. Hence, from equation (1) it can be seen that the *maximum* cell radius for a particular mobile terminal is a constant multiplied by $[1/(\eta_F+1)]^{(1/\gamma)}$. The statistics of the uplink INR η_F thus enable cell radius statistics to be computed.

In [19] a Monte Carlo simulation was performed to numerically estimate the cell radius statistics for comparably loaded CT2, DECT, GSM and AMPS systems. These simulation results showed that microcellular systems can suffer interference domination of wanted links up to 100 times greater than the interference domination of wanted links in large cell systems. This caused the *spread* of cell sizes in the microcell networks to be up to 10 times larger than those seen in the large cell systems.

This spread of cell sizes may cause significant coverage gaps and handoff difficulties in a microcell network. In order to progress towards an engineering methodology for a microcellular network, cell size variability needs to be examined analytically.

IV. η_F STATISTICS FOR AN INTERFERER PAIR

Equation (3) indicates the statistics of η_F are dependent upon the statistics of P_{FijF00} and P_{MijF00} (the spill powers), $r_{i,0}$ (a constant for each i) and $r_{ij,0}$. The statistics of $r_{ij,0}$ can be computed exactly for certain distributions of mobile terminals.

Consider a mobile terminal distribution where each microcell of nominal radius r contains the same number of mobile terminals, uniformly but randomly distributed throughout the cell, as shown in figure 2. The mobile terminal M_{ij} is communicating with fixed station F_i and is interfering with M_{00}'s uplink with F_0 due to mobile–to–fixed terminal channel spill P_{MF}. For notational simplicity, the interfering mobile range $r_{ij,0}$ will be called d (and the random variable to which this value belongs D) and the fixed station separation $r_{i,0}$ will be called s.

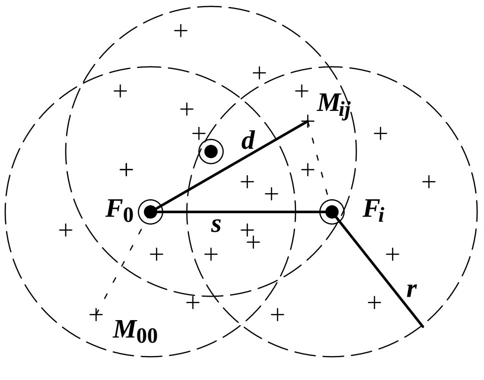

Figure 2 – Microcell terminal distribution model

Over the *service area* this terminal distribution model results in a higher density of mobiles where cells overlap (i.e. base stations are closely spaced) and lower where they don't. As service providers will only place microcell base stations closer together when there is a higher density of users to create the demand, this distribution may be a good approximation to actual non–uniform mobile terminal distribution in a microcell network.

It can be shown that when $r \le s$ the PDF of d is given by:

$$f_D(d) = \frac{2d}{\pi r^2} \arccos\left[\frac{d^2 + s^2 - r^2}{2ds} \right] \qquad s - r \le d \le s + r \tag{5}$$

and when $r > s$ the PDF of d is given by:

$$f_D(d) = \begin{cases} \dfrac{2d}{r^2} & 0 \le d \le r - s \\ \dfrac{2d}{\pi r^2} \arccos\left[\dfrac{d^2 + s^2 - r^2}{2ds}\right] & r - s < d \le s + r \end{cases} \qquad (6)$$

For a *pair* of fixed stations, each communicating with one mobile station, η_F is given by:

$$\eta_F = \frac{\kappa}{N}\left[P_{FF}s^{-\gamma} + P_{MF}d^{-\gamma}\right] \qquad (7)$$

where P_{FF} is the fixed–to–fixed station channel spill, which may be very small depending upon the system technology.

The PDF of η_F can be derived by performing a probability transformation on the PDF of d, assuming P_{FF} and P_{MF} are constants. For the mobile–fixed station interferer pair, when $r \le s$, the PDF of η_F can be shown to be :

$$f_H(\eta_F) = \frac{2N}{\pi r^2 \kappa \gamma P_{MF}} \Lambda^{\gamma+2} \arccos\left[\frac{\Lambda^2 + s^2 - r^2}{2\Lambda s}\right] \qquad (8)$$

where

$$\Lambda = \left\{\frac{1}{P_{MF}}\left[\frac{N\eta_F}{\kappa} - P_{FF}s^{-\gamma}\right]\right\}^{-1/\gamma} \qquad (9)$$

and when $r > s$, the PDF of η_F can be shown to be:

$$f_H(\eta_F) = \begin{cases} \dfrac{2N}{\pi r^2 \kappa \gamma P_{MF}} \Lambda^{\gamma+2} \arccos\left[\dfrac{\Lambda^2 + s^2 - r^2}{2\Lambda s}\right] & \eta_F < \eta_p \\ \dfrac{2N}{r^2 \kappa \gamma P_{MF}} \Lambda^{\gamma+2} & \eta_F \ge \eta_p \end{cases} \qquad (10)$$

where the point of piecewise continuity η_p is given by:

$$\eta_p = \frac{\kappa}{N}\left[P_{FF}s^{-\gamma} + P_{MF}(r - s)^{-\gamma}\right] \qquad (11)$$

The PDF in equation (10) generally exhibits a peak at low values of η_F, with a long tail that extends rapidly towards infinity as r approaches s.

The theoretical η_F PDF can be compared with a Monte Carlo simulation on a microcellular network comprising two fixed stations, each communicating with one portable terminal. The Monte Carlo simulation program [18],[19] places the mobile terminals randomly but with a uniform distribution within each cell, as per the terminal distribution model (figure 2).

Consider a CT2 network which is perfectly synchronised (i.e. all base stations transmit at exactly the same time, so there is no fixed–to–fixed station interference) with the simulation parameters as shown in table 1.

TABLE 1 – CT2 SIMULATION PARAMETERS

Parameter	Value
γ	3.5
κ	7.589×10^{-4}
N	–111.0 dBm
P_{FF}	0.0 W
P_{MF}	1.259×10^{-7} W
r	100 m
s	100 m

A channel spill P_{MF} of –49 dBc (126 nW) represents a situation where the DCA algorithm always maintains 3 or more RF channels between the two CT2 links. Simulations performed on this basis are denoted the ‘equal–spill simulations’ (EqS).

In practice, some terminals may use closer channels (even under DCA) that spill greater amounts of power and thus create more interference. Simulations using DCA and the exact spills as per the minimum ETSI requirement for CT2 [21] are denoted the ‘exact spill simulations’ (ExS). These simulations will demonstrate the effect of the equal spill simplification.

Simulated values of η_F for *successful* calls were collected for 10000 random static call attempts in accordance with terminal distribution model. Figure 3 compares the EqS PDF with the theoretical PDF produced by substituting the system parameters as per table 1 into equation (8). Figure 3 shows good agreement between the theoretical and simulated results.

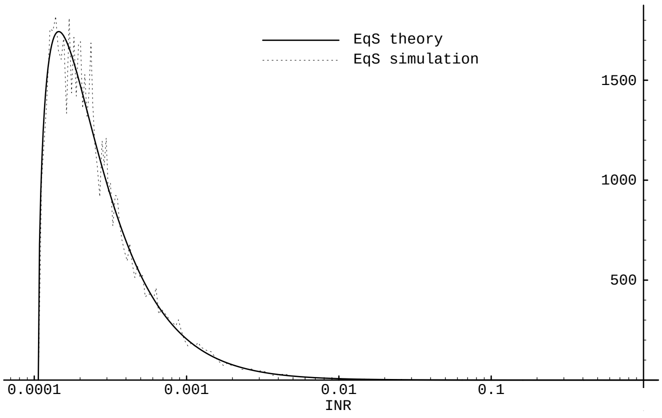

Figure 3 – η_F PDF for a CT2 interferer pair

The η_F CDFs, however, are of greater interest as the upper tail of the η_F distribution will determine what proportion of mobile terminals will experience below–target cell sizes. Figure 4 compares the theoretical and simulated (EqS and ExS) CDFs for a CT2 interferer pair. The CDF is plotted on a lognormal scale to amplify the tails of the distribution.

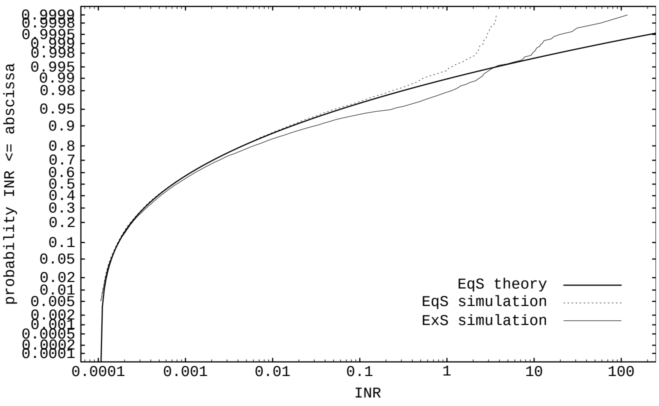

Figure 4 – η_F CDFs for a CT2 interferer pair

Figure 4 shows that the agreement between the simulated and theoretical CDFs is good. However, the EqS simulation does not agree exactly with the theoretical CDF at the upper tail of the distribution because the simulation knocks out calls which fail to meet S/[N+I] requirements whilst equation (8) assumes all calls succeed. This has the effect of skewing the tail of the distribution towards lower values of η_F as terminals with higher INRs are more likely to fail.

The ExS simulation still compares well with the EqS theory. With the exact spills, near–adjacent channel transmissions will now cause greater interference than assumed in the EqS theory. This has the effect of slightly reducing the probability of low η_F values in the middle of the distribution. The η_F distribution is again skewed at the upper tail of the distribution due to call loss, but at higher η_F values than before.

Interestingly, the call loss rate in the ExS simulation (0.57%) was only a little higher than the call loss rate in the EqS simulation (0.45%).

The cell radius CDFs can be produced directly from the η_F CDFs by using equation (1) as shown in figure 5. The cell radius CDFs are plotted on a log probability scale to amplify the distribution at the lower tail, which determines the proportion of mobile terminals experiencing below target cell sizes.

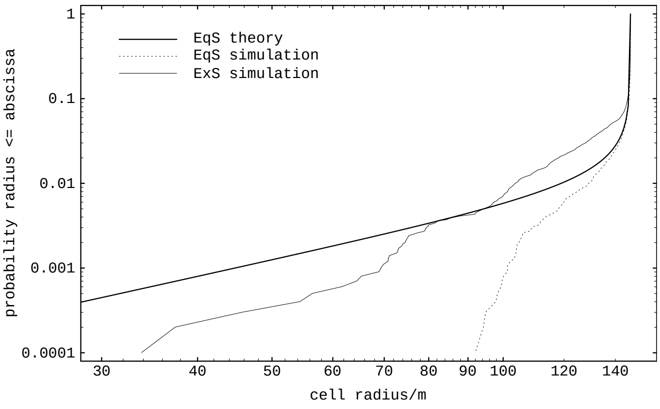

Figure 5 – Cell radius CDFs for a CT2 interferer pair

Examining figure 5, it appears that the theoretical cell radius CDF is pessimistic in comparison to the EqS simulation result. Theory predicts that 1% of mobile terminals in this network will have cell radii smaller than 119 metres, but the EqS simulation predicts only 0.6% of terminals will have cell radii smaller than this. With the exact spills, the theoretical result is optimistic, with the ExS simulation indicating approximately 2.1% of mobiles achieve cell radii smaller than 119 m.

V. η_F STATISTICS FOR AN INTERFERER ENSEMBLE

To extend the results of the previous section to an interferer ensemble exactly may not be tractable. However, it may be possible to introduce simplifying assumptions.

If an infinite ensemble of interferers is assumed, it may be possible to apply the Central Limit Theorem to the known PDF for an interferer pair. The Central Limit Theorem states that the distribution of an infinite number of identically distributed random variables approaches a Gaussian distribution. The η_F PDF will have the same functional form as derived previously for each additional interferer, but the *r*/*s* ratio will change depending upon the fixed station arrangement. It might be expected that the distribution of η_F would become more and more Gaussian–like the larger the interference field.

This can be tested numerically with the Monte Carlo model. The η_F statistics for a regular arrangement of CT2 cells were computed for 1, 3 and 5 tiers of cells (7, 37 and 91 cells respectively) with 1 user in each cell and all other parameters as per table 1 (except the ETSI CT2 channel spills were used). The distance *s* between each fixed station was kept at 100m, so this represents a larger amount of cell overlap than traditional hexagonal cell clusters. The resultant η_F and cell radius CDFs are shown in figures 6 and 7.

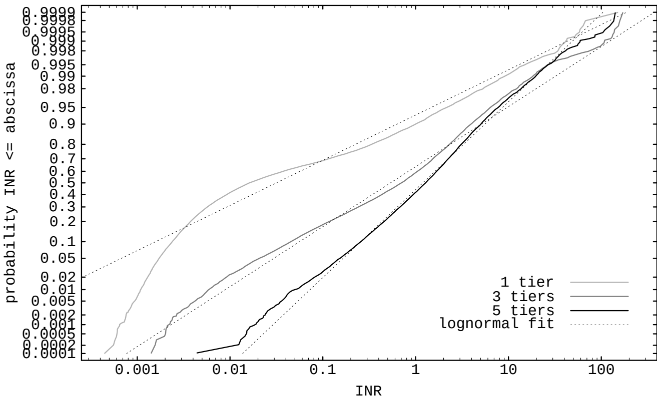

Figure 6 – η_F CDFs for 1, 3 and 5 tiers of CT2 cells

Figure 6 strongly suggests that the η_F distribution becomes more Gaussian as the number of interferers increases. The cell radius CDF (figure 7) shows that the lognormal INR approximation accurately predicts cell radii down to better than the last percent of terminals for the 5 tier simulation. If application of the CLT is possible, then the cell radius statistics for an interferer ensemble, and thus coverage quality, could be described in terms of the fundamental system design parameters.

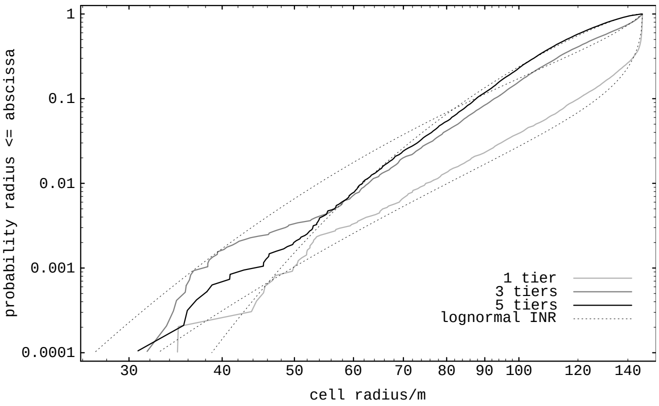

Figure 7 – Cell radius CDFs for 1, 3 and 5 tiers of CT2 cells

VI. CONCLUSION

Cell sizes experienced by mobile stations in a cellular system can be modelled as a function of their uplink Interference to Noise ratio (INR) η_F. The cell size statistics affect the probability of obtaining contiguous radio coverage for a certain proportion of mobile stations.

The statistics of the INR can be derived analytically under simplifying assumptions, and show good agreement with Monte Carlo simulations where these simplifying assumptions are not made. It is hypothesised that the Central Limit Theorem may be applied to approximate the behaviour of an interferer ensemble, which would enable the cell size statistics for a network to be expressed in terms of the system design parameters.

Further development of the analysis approach described in this paper is expected to form the basis for a systematic microcellular engineering methodology.

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