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# COCHANNEL REUSE DISTRIBUTIONS IN DCA MICROCELLULAR SYSTEMS WITH IN–CELL REUSE CONSTRAINT

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**ABSTRACT**

This paper presents a novel analysis of cochannel reuse in Dynamic Channel Assignment (DCA) microcellular systems with an in–cell channel reuse constraint. Mathematical analysis is used to show that DCA systems exhibit significantly closer cochannel reuse than Fixed Channel Assignment (FCA) systems despite the reuse constraint. Closed form expressions for the cochannel reuse ratio (CRR) distributions in DCA microcell systems are derived. It is shown that the DCA reuse distributions cannot be obtained using conventional cellular engineering techniques, and are a fundamental consequence of the microcell architecture.

## 1 INTRODUCTION

In conventional cellular radio communications systems ('macrocell' systems), the available radio channels are partitioned into $C$ channel sets ($C$ is called the 'cluster size') and each transmitter is allocated the use of one of these channel sets [1]. This is called Fixed Channel Assignment (FCA). Once the system grows beyond $C$ cells, the channel sets are reused in such a way as to not cause excessive interference to existing cells.

The channel reuse ratio in a cellular system is defined as the distance $d$ between cells using related channels divided by the cell radius $r$. If a pair of terminals in two cells are using the same channel, the ratio $d/r$ is called the cochannel reuse ratio (CRR). If a pair of terminals in the two cells are using immediately adjacent channels, the ratio $d/r$ is called the adjacent channel reuse ratio (ACRR).

The capacity of macrocellular systems can be increased by splitting existing cells into smaller cells, reusing frequencies more often in a geographic area but keeping $d/r$ constant. In practice, however, there is a capacity limit as cells cannot be split indefinitely. The lower cell radius limit for most macrocell systems is in the range of 1 to 1.5 km [2].

Microcellular technologies are being developed to provide wireless communications to very large numbers of people at a much higher user density than is possible with macrocells [3]. Microcellular architecture differs from macrocell architecture in three fundamental ways:

- The cells have a much smaller radius
- The mobile terminals transmit at much lower power levels
- All radio channels are available in every cell

It is generally impractical in a microcell system to preassign channels using FCA. Instead, channels are assigned at call set up time by the mobile terminal or base station, with the aim of the channel assignment algorithm being the minimisation of interference. This is called Dynamic Channel Assignment (DCA).

In FCA systems there is a simple relationship between the cluster size $C$, the CRR $d/r$, and the signal to interference ($s/i$) performance of a receiver at a cell boundary in the presence of cochannel interferers [1],[4]–[7]. However, no such simple relationship between cluster size, $d/r$, and worst case $s/i$ performance exists for microcells [7].

This paper derives the CRR distribution in a DCA microcell system where an in–cell cochannel reuse constraint exists, i.e. there is a limit to the closest approach of a cochannel interferer. This analysis establishes a theoretical limit to CRRs in DCA microcell systems. This result provides a basis for predicting the closest approach of interferers and the worst case $s/i$ performance for microcells, and therefore the quality of the radio coverage offered by the microcell system.

## 2 COCHANNEL REUSE RATIOS

### 2.1 Cochannel Reuse in DCA Systems

Cochannel reuse within DCA and FCA systems is shown in an idealised way in Fig. 1. Each cell is represented as a hexagon, although in practice cells have irregular boundaries. In the FCA system, the available channels are divided into $C$ sets (in Fig. 1 $C = 3$) and the nominal CRR is given by $\sqrt{3C} = 3.0$. As cochannel mobile terminals are confined to the cochannel cell, it can be seen from Fig. 1 that the minimum possible CRR for a terminal in this particular FCA system is 2.0.

In FCA systems, the $s/i$ requirement of the particular technology determines the minimum cluster size and the channel allocation pattern. This design principle, however, breaks down in microcells because there is no channel partitioning in DCA systems. Every terminal has the capability of using any channel in any cell. The reuse ratio probabilities cannot be predicted using macrocell design principles [8] and the minimum CRR in a microcell systems would be a complicated function of the terminal distribution, server access rules, channel assignment algorithm, and previous channel selections.

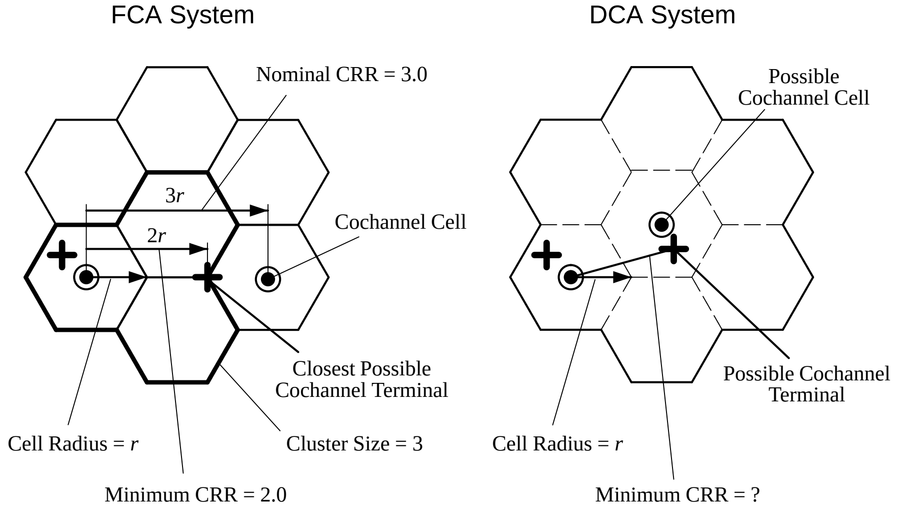


**Figure 1 – Cochannel Reuse Ratio in FCA vs DCA Systems**

Researchers often do not examine CRR probabilities, despite the fact that terminals are randomly located. For example, Linnartz [9] assumed all interfering terminals in an FCA system were located at the nominal reuse distance; Wang and Rappaport [10],[11] assumed terminals were in the 'worst case' location in each cell; and Chuang [12] assumed terminals were located at regular fixed points throughout the service area. Whilst this may be acceptable for an FCA system, it is not clear that it is appropriate for DCA systems.

### 2.2 Monte Carlo Simulation

A computer program was developed to model arbitrary cellular networks [13]–[17] and perform Monte Carlo simulations to estimate system performance parameters such as CRR and ACRR probabilities. In [18] CRR and ACRR simulation results were presented for systems of 21 cells arranged in a regular, hexagonal pattern. Four mobile technologies were simulated: GSM (the most widely used digital macrocell system); CT2 (a second generation digital cordless telephone system); DECT (the Digital European Cordless Telephone system); and PHS (the Japanese Personal Handy Phone System).

The results in [18] showed that the microcell systems (CT2, DECT and PHS) exhibited significantly closer channel reuse than the macrocell system (GSM). In the case of cochannel reuse, the mean CRR was lower for GSM than the microcell systems, but the *minimum* CRR was much lower in the microcell systems – as small as 1.2, compared with 2.0 for GSM.

Further, when the distribution of CRR values was plotted, it showed that a significant *proportion* of microcell terminals successfully reused cochannels at small CRRs. The simulation results showed that 2.1% of cochannel CT2 terminals, 3.0% of cochannel PHS terminals, and 4.5% of cochannel DECT terminals successfully operated at CRRs of less than 2.0.

The significance of the CRR distributions was illustrated by plotting the physical locations of the cochannel interferers with respect to the nominal, idealised, cell boundaries [18]. Fig. 2 shows the location of the first 100 cochannel interferers to users in the central reference cell for the four simulations in.

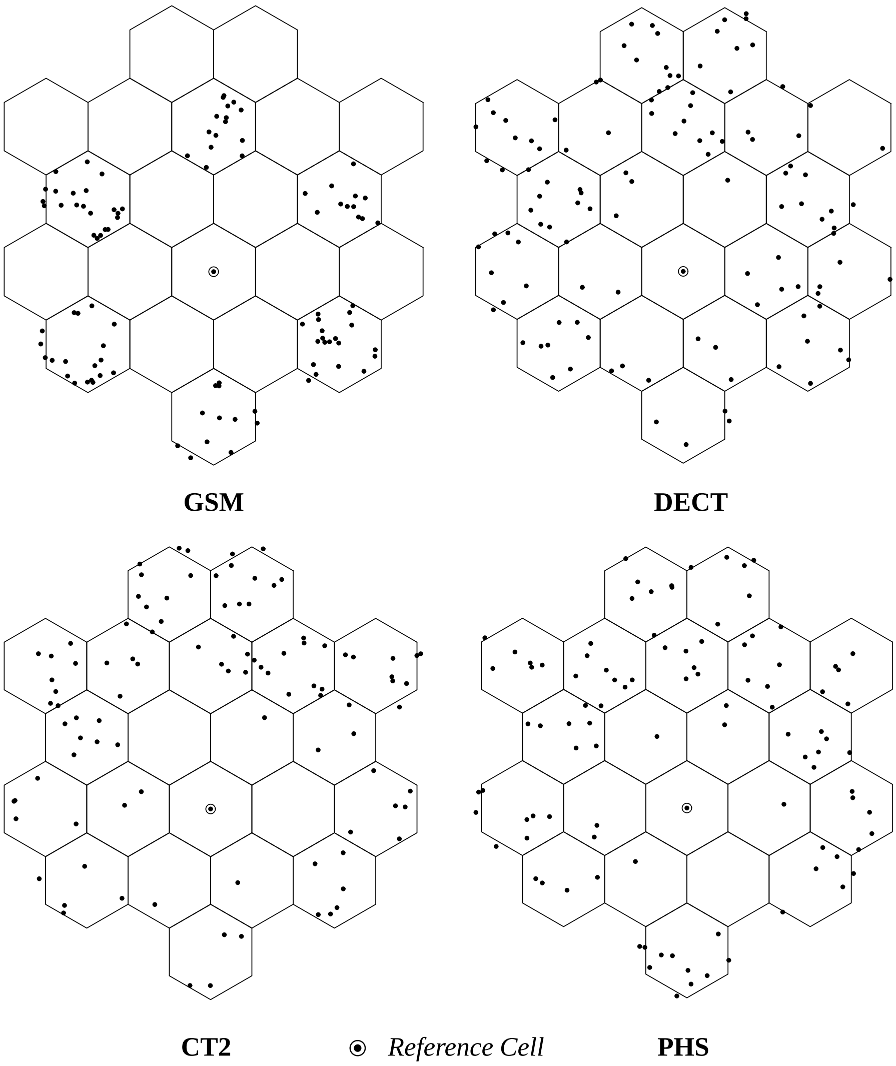


**Figure 2 – Location of cochannel interferers for four technologies [18]**

The significance of the different cochannel reuse patterns are clearly apparent in Fig. 2. Firstly, the operation of FCA in GSM permits cochannel use only in the six designated first tier cells and prevents cochannel use in cells adjacent to the reference cell. In the DCA microcell systems, however, cochannel interferers establish themselves in cells adjacent to the reference cell. This can cause significant coverage loss because close cochannel interferers severely limit the range of affected terminals [13]–[16].

## 3 COCHANNEL REUSE RATIO DISTRIBUTION ANALYSIS

### 3.1 Channel Reuse Ratio Model

The factors which influence the channel reuse ratio probabilities include:

- The terminal distribution
- The cell layout and service area extent
- The channel assignment algorithm
- The propagation model

By making simplifying assumptions and following the channel reuse model as shown in Fig. 3, the channel reuse ratio probabilities may be derived analytically. The propagation model assumed for the following derivation is a single exponent distance–dependent path loss model of the form $P_r \propto P_t\, d^{-\gamma}$. Shadow fading was not considered for reasons of analytical tractability. Additionally, only a single cochannel interferer is assumed, however this provides a lower bound to the CRR probabilities.

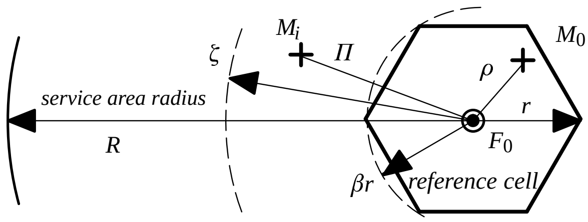


**Figure 3 – Channel reuse model**

To calculate the reuse probabilities, the *a priori* assumption is that a cochannel reuse event has occurred, hence the terminal $M_0$ must be in the reference cell (radius $r$) at some radius $\rho$ from the reference cell site $F_0$. The interfering cochannel terminal $M_i$ is assumed to be within the service area annulus between $\beta r$ and $R$ and at radius $\Pi$ from $F_0$.

CRR and ACRR distributions were derived in [18], however, the CRR derivation had no in–cell cochannel reuse constraint, i.e. it placed no lower limit on where potential cochannel terminals could be located in space.

In practice, cell site equipment always prevents cochannel reuse within the one cell regardless of whether DCA or FCA is used. Should a cochannel interferer $M_i$ in an adjacent cell move within the boundary of the reference cell, a handover to the reference cell would probably occur and a new channel would be assigned. This has the effect of placing a lower bound on the spatial location of potential cochannel interferers. This is shown as the inner radius $\beta r$ in Fig. 3 ($\beta$ is a dimensionless multiplicative factor).

The signal received at $F_0$ from $M_0$ is denoted as $s$, the interference received at $F_0$ from $M_i$ as $i$, and the signal to interference ratio $s/i$ as $z$, with the random variable (RV) to which it belongs $\mathfrak{Z}$. The CRR is given by $\Re = \Pi/r$ and the RV to which it belongs is denoted $\mathfrak{R}$. For DCA systems the problem is to compute the conditional distribution of the CRR:

$$F_{\mathfrak{R}}\left(\Re \;\middle|\; z \ge Z\right) = F_{\mathfrak{R}}\left(\Re \;\middle|\; \frac{s}{i} \ge Z\right) = \frac{p\left\{\frac{\Pi}{r} \le \Re \,,\; \frac{s}{i} \ge Z\right\}}{p\left\{\frac{s}{i} \ge Z\right\}} \tag{1}$$

where $p\{x\}$ is the probability of event $x$ occurring, and $Z$ is the required signal to interference protection ratio for the particular technology.

Note that receiver noise $n$ has been omitted from Eq. (1), and hence only a $s/i$ threshold rather than a $s/[n+i]$ threshold is in force. There are two reasons for this. Firstly, cochannel interferers are much stronger than receiver noise by many orders of magnitude unless the interferer is at a great distance. As the critical area of interest is the probability of achieving *small* reuse ratios, and any error resulting from the assumption that $n = 0$ should be necessarily vanishing. Secondly, if receiver noise $n$ is included an exact closed form analysis is not possible.

The significance of the radius $\zeta$ in Fig. 3 is as follows. There will be some range $\Pi > \zeta$ at which the interferer $M_i$ cannot generate sufficient interference at $F_0$ to cause $M_0$'s link to fail regardless of $M_0$'s location. Thus when $\Pi > \zeta$, the signal to interference ratio $z$ will always be greater than or equal to $Z$ and the probabilities $p\{\Pi/r \le \Re\}$ and $p\{s/i \ge Z\}$ in the numerator of Eq. (1) become independent. Conversely, for $\Pi \le \zeta$ the probabilities in the numerator of Eq. (1) are *not* independent and therefore the computation of $p\{\Pi/r \le \Re , s/i \ge Z\}$ becomes more complicated.

The value of $\zeta$ may be easily computed under the assumed propagation model. With this model $s = \kappa P_t \rho^{-\gamma}$ and $i = \alpha\kappa P_t \Pi^{-\gamma}$, where $\alpha$ is the relative strength of the interferer (for a cochannel interferer $\alpha = 1$), $\kappa$ is an RF constant, $P_t$ is the transmit power and $\gamma$ is the path loss exponent. The minimum possible signal power occurs when $M_0$ is at the periphery of its cell, i.e. $s = \kappa P_t r^{-\gamma}$, hence the minimum $s/i$ ratio for $M_0$ is given by:

$$z = \frac{s}{i} = \frac{\kappa P_t r^{-\gamma}}{\alpha\kappa P_t \Pi^{-\gamma}} = \frac{1}{\alpha}\left(\frac{\Pi}{r}\right)^{\gamma} = \frac{1}{\alpha}\left(\frac{r\Re}{r}\right)^{\gamma} = \frac{\Re^{\gamma}}{\alpha} \tag{2}$$

When $z = Z$,

$$\zeta = r\Re = r(\alpha Z)^{1/\gamma} \tag{3}$$

hence the conditional distribution of Eq. (1) is piecewise continuous about a reuse ratio $\Re = (\alpha Z)^{1/\gamma}$. It can be shown that when $\Re \le (\alpha Z)^{1/\gamma}$, Eq. (1) can be written as:

$$\frac{p\left\{\frac{\Pi}{r} \le \Re \ , \ \frac{s}{i} \ge Z\right\}}{p\left\{\frac{s}{i} \ge Z\right\}} = \frac{F_{\mathfrak{R}}(\Re) - F_{\mathfrak{R}\mathfrak{Z}}(\Re, Z)}{1 - F_{\mathfrak{Z}}(Z)} \qquad \beta \le \Re \le (\alpha Z)^{1/\gamma} \tag{4}$$

where $F_{\mathfrak{R}\mathfrak{Z}}(\Re, Z)$ is the joint distribution of $\mathfrak{R}$ and $\mathfrak{Z}$. When $\Re > (\alpha Z)^{1/\gamma}$, Eq. (1) needs to be reformulated, and it can be shown that the required distribution becomes:

$$F_{\mathfrak{R}}\left(\Re \ \middle| \ z \ge Z \ , \ \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) = \frac{F_{\mathfrak{R}}(\Re) - F_{\mathfrak{R}}\left((\alpha Z)^{1/\gamma}\right)}{1 - F_{\mathfrak{R}}\left((\alpha Z)^{1/\gamma}\right)} \qquad (\alpha Z)^{1/\gamma} < \Re < \frac{R}{r} \tag{5}$$

Note that Eq. (5) is a distribution function in its own right, based upon the assumption that $\Re > (\alpha Z)^{1/\gamma}$. To obtain the *a priori* distribution, Eq. (5) must be scaled by $(1–q)$ and shifted by $q$, where $q$ is the value of Eq. (4) at the reuse ratio breakpoint $\Re = (\alpha Z)^{1/\gamma}$.

### 3.2 DCA Cochannel Reuse Ratio Distribution Derivation

As indicated earlier, in the following derivation it will be assumed that only *one* interferer exists for any particular cochannel reuse event, and that it is the dominant interference source (i.e. other interferers and receiver noise $n$ are negligible). For a cochannel interferer $\alpha = 1$, however this parameter will be retained in the derivations for generality.

Examining Eq. (4), the first expression requiring evaluation is $F_{\mathfrak{R}}(\Re)$. If it is assumed that $M_0$ is distributed within the reference cell (radius $r$) uniformly by area, and that $M_i$ is distributed within the annulus between $\beta r$ and $R$ uniformly by area, it can be shown that:

$$F_{\Re}(\Re) = \frac{r^2\Re^2 - \beta^2 r^2}{R^2 - \beta^2 r^2} \qquad \beta \le \Re \le \frac{R}{r} \tag{6}$$

Next, the joint distribution function $F_{\mathfrak{R}\mathcal{Z}}(\Re, Z)$ is defined as:

$$F_{\Re\mathcal{Z}}(\Re, Z) = \int_{\beta}^{\Re}\int_{\alpha^{-1}\mathbb{R}^{\gamma}}^{Z} f_{\mathcal{Z}}\left(z \,\middle|\, \mathfrak{R} = \mathbb{R}\right) f_{\Re}(\mathbb{R})\ dz\ d\mathbb{R} \tag{7}$$

where $\mathbb{R} \in \mathfrak{R}$ is a dummy variable. The density function $f_{\mathfrak{R}}(\Re)$ is simply the derivative, with respect to $\Re$, of the distribution function $F_{\mathfrak{R}}(\Re)$ given in Eq. (6). Hence:

$$f_{\Re}(\Re) = \frac{2r^2\Re}{R^2 - \beta^2 r^2} \qquad \beta \le \Re \le \frac{R}{r} \tag{8}$$

The conditional density function $f_{\mathcal{Z}}(z \mid \mathfrak{R} = \mathbb{R})$ is the density function of the signal to interference ratio $z$ given a specific reuse ratio and thus a specific amount of interference. Under these conditions the minimum possible value of $z$ is $\alpha^{-1}\mathbb{R}^{\gamma}$. Given the assumed distributions of $M_0$ and $M_i$ the density functions of the signal and interference powers can be shown to be:

$$f_S(s) = \frac{2}{\gamma r^2}(\kappa P_t)^{\frac{2}{\gamma}}\ s^{-\frac{\gamma+2}{\gamma}} \qquad \kappa P_t r^{-\gamma} \le s < \infty \tag{9}$$

$$f_I(i) = \frac{2}{\gamma\left(R^2 - \beta^2 r^2\right)}(\alpha\kappa P_t)^{\frac{2}{\gamma}}\ i^{-\frac{\gamma+2}{\gamma}} \qquad \alpha\kappa P_t R^{-\gamma} \le i < \alpha\kappa P_t(\beta r)^{-\gamma} \tag{10}$$

With appropriate transformations, the required conditional density function is given by:

$$f_{\mathcal{Z}}\left(z \,\middle|\, \mathfrak{R} = \mathbb{R}\right) = \frac{2\alpha\mathbb{R}^2}{\gamma}\ (\alpha z)^{-\frac{\gamma+2}{\gamma}} \qquad \frac{\mathbb{R}^{\gamma}}{\alpha} \le z < \infty \tag{11}$$

thus the joint distribution function as per Eq. (7) may be evaluated to be:

$$F_{\Re\mathcal{Z}}(\Re, Z) = \frac{r^2}{\alpha^{\frac{2}{\gamma}}\left(R^2 - \beta^2 r^2\right)}\left[\alpha^{\frac{2}{\gamma}}\left(\Re^2 - \beta^2\right) + \frac{\Re^4 - \beta^4}{2Z^{\frac{2}{\gamma}}}\right] \tag{12}$$

$$\beta \le \Re \le \frac{R}{r}\ ,\ \frac{\Re^\gamma}{\alpha} \le Z < \infty$$

The density function of the signal to interference ratio $z = s/i$ is given by [19]:

$$f_{\mathcal{Z}}(z) = \int_{-\infty}^{\infty} |i| \cdot f_{SI}(zi, i)\ di = \int_{-\infty}^{\infty} |i| \cdot f_S(zi) \cdot f_I(i)\ di \tag{13}$$

as $s$ and $i$ are independent. It can be shown that $f_{\mathcal{Z}}(z)$ is piecewise continuous about $z = \alpha^{-1}(R/r)^\gamma$, with the expression:

$$f_{\mathcal{Z}}(z) = \begin{cases} \dfrac{r^2\alpha^{\frac{2}{\gamma}}}{\gamma z^{\frac{\gamma+2}{\gamma}}\left(R^2 - \beta^2 r^2\right)}\left[z^{\frac{4}{\gamma}} - \beta^4\alpha^{-\frac{4}{\gamma}}\right] & \dfrac{\beta^\gamma}{\alpha} \le z \le \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma \\ \dfrac{1}{\gamma r^2\alpha^{\frac{2}{\gamma}} z^{\frac{\gamma+2}{\gamma}}\left(R^2 - \beta^2 r^2\right)}\left[R^4 - \beta^4 r^4\right] & \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma < z < \infty \end{cases} \tag{14}$$

The distribution function $F_{\mathcal{Z}}(z)$ can be obtained by integrating Eq. (14) with respect to $z$ over the appropriate limits, giving:

$$F_{\mathcal{Z}}(z) = \begin{cases} \dfrac{r^2\left(\beta^2 - \alpha^{\frac{2}{\gamma}} z^{\frac{2}{\gamma}}\right)^2}{2\alpha^{\frac{2}{\gamma}} z^{\frac{2}{\gamma}}\left(R^2 - \beta^2 r^2\right)} & \dfrac{\beta^\gamma}{\alpha} \le z \le \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma \\ 1 - \dfrac{R^4 - \beta^4 r^4}{2r^2\alpha^{\frac{2}{\gamma}} z^{\frac{2}{\gamma}}\left(R^2 - \beta^2 r^2\right)} & \dfrac{1}{\alpha}\left(\dfrac{R}{r}\right)^\gamma < z < \infty \end{cases} \tag{15}$$

The CRR distribution for $\Re \le (\alpha Z)^{1/\gamma}$ computed from Eq. (4) is thus:

$$F_{\Re}\left(\Re \;\middle|\; z \ge Z\right) = \frac{r^2\left(\Re^4 - \beta^4\right)}{2R^2(\alpha Z)^{\frac{2}{\gamma}} - r^2(\alpha Z)^{\frac{4}{\gamma}} - r^2\beta^4} \qquad \beta \le \Re \le (\alpha Z)^{1/\gamma} \tag{16}$$

and the *a posteriori* CRR distribution for $\Re > (\alpha Z)^{1/\gamma}$ computed from Eq. (5) is:

$$F_{\Re}\left(\Re \;\middle|\; z \ge Z\ ,\ \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) = \frac{r^2\Re^2 - r^2(\alpha Z)^{\frac{2}{\gamma}}}{R^2 - r^2(\alpha Z)^{\frac{2}{\gamma}}} \qquad (\alpha Z)^{1/\gamma} \le \Re \le \frac{R}{r} \tag{17}$$

As described earlier, Eq. (17) is a distribution function in its own right, with a minimum value of zero and a maximum value of unity. As part of the overall reuse ratio distribution, the *a priori* CRR distribution for $\Re > (\alpha Z)^{1/\gamma}$ is given by:

$$F_{\Re}\left(\Re \;\middle|\; z \geq Z\right) = q + (1 - q)F_{\Re}\left(\Re \;\middle|\; z \geq Z\ ,\ \frac{\Pi}{r} > (\alpha Z)^{1/\gamma}\right) \tag{18}$$

where

$$q = F_{\Re}\left((\alpha Z)^{1/\gamma} \;\middle|\; z \geq Z\right) = \frac{r^2\left((\alpha Z)^{\frac{4}{\gamma}} - \beta^4\right)}{2R^2(\alpha Z)^{\frac{2}{\gamma}} - r^2(\alpha Z)^{\frac{4}{\gamma}} - r^2\beta^4} \tag{19}$$

i.e. the value of the distribution (from Eq. (16)) at the reuse ratio breakpoint $\Re = (\alpha Z)^{1/\gamma}$. Hence the complete CRR distribution is given by:

$$F_{\Re}\left(\Re \;\middle|\; z \geq Z\right) = \begin{cases} \dfrac{r^2\left(\Re^4 - \beta^4\right)}{2R^2(\alpha Z)^{\frac{2}{\gamma}} - r^2(\alpha Z)^{\frac{4}{\gamma}} - r^2\beta^4} & \beta \leq \Re \leq (\alpha Z)^{1/\gamma} \\[2ex] \dfrac{2r^2\Re^2(\alpha Z)^{\frac{2}{\gamma}} - r^2(\alpha Z)^{\frac{4}{\gamma}} - r^2\beta^4}{2R^2(\alpha Z)^{\frac{2}{\gamma}} - r^2(\alpha Z)^{\frac{4}{\gamma}} - r^2\beta^4} & (\alpha Z)^{1/\gamma} \leq \Re \leq \dfrac{R}{r} \end{cases} \tag{20}$$

Eq. (20) represents the theoretical CRR *limit* for a DCA system with interference protection ratio $Z$ and cells of radius $r$, operating in a service area of radius $R$ where cochannel terminals are not permitted within a range $\beta r$ of the reference cell. In practice, cochannel reuse may not approach this limit, as it was based upon an assumption that a single interferer dominated. Typically, there will be some additional interference which is not negligible [10]. For example, it has been shown that adjacent cochannel interference can affect the performance of heavily loaded systems [11],[20]–[23].

Note that Eq. (20) only applies if $R/r > (\alpha Z)^{1/\gamma}$. If $R/r = (\alpha Z)^{1/\gamma}$ then the first part of Eq. (20) covers the complete range of reuse ratios $\Re$ (i.e. $\beta \leq \Re \leq R/r$). If $R/r \leq (\alpha Z)^{1/\gamma}$ a different formulation of the original probability expression, i.e. Eq. (4), is required. It can be shown that when $R/r \leq (\alpha Z)^{1/\gamma}$, the CRR distribution is given by:

$$F_{\Re}\left(\Re \;\middle|\; z \geq Z\ ,\ \frac{R}{r} \leq (\alpha Z)^{1/\gamma}\right) = \frac{r^4\left(\Re^4 - \beta^4\right)}{R^4 - r^4\beta^4} \qquad \beta \leq \Re \leq \frac{R}{r} \tag{21}$$

This is a somewhat counter–intuitive result, and is surprising for the fact that it depends upon neither $Z$ nor $\gamma$, but solely the geometry of the system ($\beta$, $r$ and $R$). When $R/r \leq (\alpha Z)^{1/\gamma}$, the entire system is inside of the radius $\zeta$ (as per Fig. 3). The probability of successful cochannel reuse at a given reuse ratio and the *maximum* possible reuse ratio decrease by the *same proportion*, hence the *shape* of the distribution remains unchanged.

Note that this does not provide information about *how likely* a cochannel reuse event is in comparison to non–cochannel reuse events. As the system shrinks, there will be fewer cochannel events, but the CDF of the cochannel events that do occur will follow Eq. (21). For typical values of $Z$ (less than 20 dB) and systems comprising more than a few cells, $R/r > (\alpha Z)^{1/\gamma}$, rendering this result little more than a curiosity.

### 3.3 DCA CRR Distribution Comparison With FCA CRR Distribution

To compare the DCA CRR distribution with the FCA CRR distribution, the FCA result from [18] may be used. The FCA CRR distribution is given by:

$$F_{\Re}(\Re) = 1 + \frac{1}{\pi}\left\{ \Re^2 \cos^{-1}\left[\frac{r^2(\Re^2 - 1) + s^2}{2rs\Re}\right] - \cos^{-1}\left[\frac{r^2(\Re^2 - 1) - s^2}{2rs}\right] - \frac{s}{r}\sqrt{1 - \left(\frac{r^2(\Re^2 - 1) - s^2}{2rs}\right)^2}\right\} \qquad \frac{s - r}{r} \le \Re \le \frac{s + r}{r} \tag{22}$$

where $s$ is the distance between the centres of the cochannel cells. The difference between the CRR distributions for DCA systems (Eq. (20)) and FCA systems (Eq. (22)) is evident. The distribution of Eq. (20) is a fundamental consequence of the lack of channel partitioning in DCA. This result proves that DCA CRR distributions cannot be predicted using conventional cellular engineering techniques.

### 3.4 DCA CRR Comparison With and Without In–cell Reuse Constraint

The impact of the in–cell reuse constraint on the DCA CRR distributions can be illustrated by comparing Eq. (20) with the distribution derived in [18] which had no in–cell reuse constraint. The result is shown in Fig. 4 for three microcell technologies (DECT, PHS and CT2) for a network of 21 cells with $\gamma$ = 3 and $\beta$ = 1.

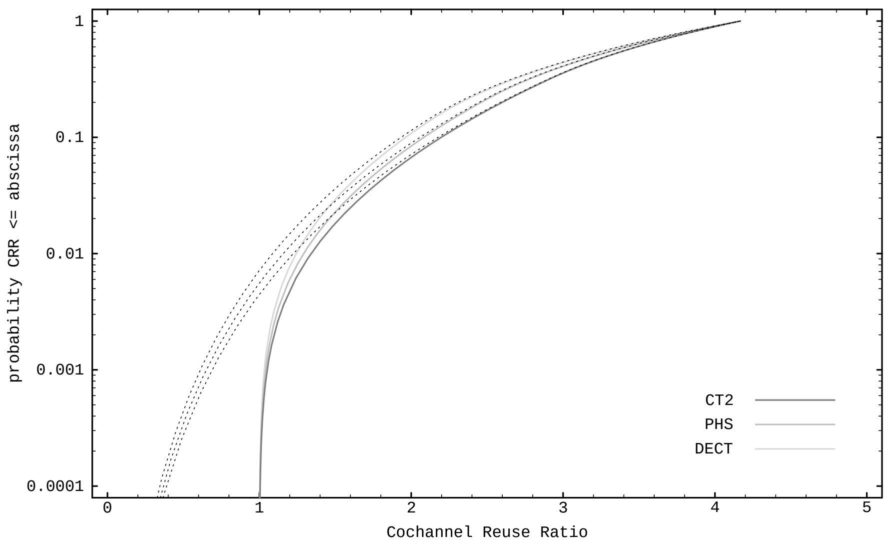


**Figure 4 – Comparison between DCA CRR distributions with (solid lines) and without (dashed lines) in–cell reuse constraint. 21 cells, $\gamma$ = 3, $\beta$ = 1.**

It can be seen that below the first percentile, the CRR distributions begin to substantially diverge. The probability of CRRs below 1.2 are significantly higher in the absence of the in–cell reuse constraint. Additionally the CRR can reach arbitrarily small values in the absence of the in–cell reuse constraint, whereas the fundamental lower CRR limit with the in–cell reuse constraint is given by $\beta$ (unity in this case).

### 3.5 Theoretical Comparison with Monte Carlo Simulation

The theoretical DCA CRR distribution derived in Sec. 3.2 and given by Eq. (20) can be compared with the Monte Carlo simulation results of [18] to assess the degree to which the simplifying assumptions reduce the accuracy of the results.

The DCA CRR distribution requires the service area radius $R$ to be computed. As an approximation, the service area comprising hexagonal cells may be replaced with a circle of radius $R$ of equal area, centred on the reference cell [18]. For $T$ tiers of a $C$ cluster hexagonal cell system this radius equals [18]:

$$R = \sqrt{\frac{3\sqrt{3}\,r^2 C(1 + 3T + 3T^2)}{2\pi}} \tag{23}$$

Firstly, the systems modelled comprised 1 tier of a (nominal) 3 cell cluster, with each cell 100 m in radius. Thus $C = 3$ and $T = 1$ and from Eq. (23) the radius of the equivalent service area is $R = 416.7$ m.

Next, the interference protection ratio $Z$ is 14 dB for CT2 [24], 12 dB for PHS [25], and 10 dB for DECT [26]. To evaluate the CRR distribution, $\alpha = 1$ in all cases as cochannel interferers are of equal strength to the signal source. In each simulation the path loss exponent $\gamma$ was set to 3.0, thus for all three systems $Z \leq \alpha^{-1}(R/r)^{\gamma}$ is satisfied and therefore Eq. (20) is applicable.

The parameter $\beta$ was set to unity to represent the situation where a potential cochannel interferer is always handed over to the reference cell exactly at the reference cell boundary. Depending upon the handover parameters in a live network, the value of $\beta$ will exhibit statistical fluctuation and may have a mean different to unity.

For the GSM simulation of [18], the required parameters are $r = 1$ km, $s = 3.0$ km, $Z = 9$ dB [27] and $\gamma = 3.0$. The minimum possible CRR of 2.0 is greater than $Z^{1/\gamma}$ and hence Eq. (22) applies. The DCA CRR distributions were plotted and superimposed upon the Monte Carlo simulation results of [18]. The result is shown in Fig. 5.

Firstly, it can be seen in Fig. 5 that the CRR distributions for all microcell technologies are asymptotic to $\Re = 1$ due to the in–cell cochannel reuse constraint. This results in a much more accurate prediction of the probabilities of small CRRs than seen in [18].

Secondly, the effect of the assumption of only one dominant cochannel interferer can be seen. The Monte Carlo curves for all the microcell technologies are inside the theoretical curves due to additional, unaccounted for, interference. This has the effect of reducing the probability of successful link establishment at all reuse ratios. This means that the theoretical curves represent the absolute *lower limit* to the CRR in each case, as the minimum amount of cochannel interference is that caused by a single interferer.

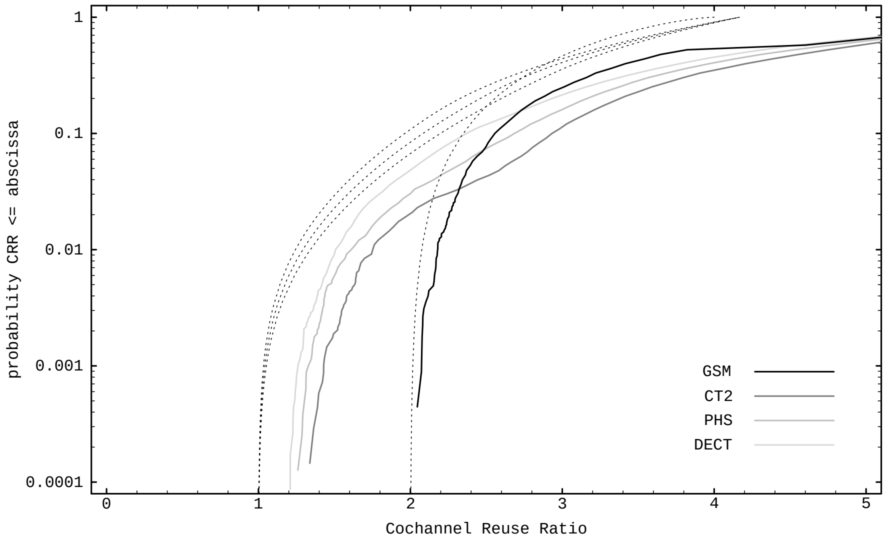


**Figure 5 – Theoretical CRR distributions (dotted lines) compared with Monte Carlo results (L to R the theoretical curves are for DECT, PHS, CT2 and GSM). 21 cells, $\gamma = 3$, $\beta = 1$.**

Eq. (20) therefore enables analytical determination of the minimum possible reuse ratio for a given proportion of cochannel terminals in a DCA microcell system. This is an important result and will assist in the characterisation of radio coverage performance in microcell systems. The accuracy of Eq. (22) for FCA systems is very good and is a consequence of the fundamental design principles used in FCA systems.

Note that the maximum CRR for the theoretical distributions is $R/r$ (i.e. 4.17 for the current examples) whilst in the simulations CRRs much greater than this occurred. This is a consequence of the fact that the Monte Carlo simulation sampled *all* cochannel events, not just those in relation to the reference cell. Whilst this affects the accuracy of the theory at large CRRs, it has little effect at small CRRs, which is the area of interest.

The ACRR distribution results in [18] remain valid unless an in–cell adjacent channel reuse constraint is applied, which is generally not the case in DCA systems. If an in–cell adjacent channel reuse constraint was applied in the DCA case, the FCA and DCA distributions would become almost identical. As the effect of interferers diminish, the channel reuse ratio distributions begin to be dominated by the system geometry rather than RF interference.

## 4 CONCLUSION

The CRR distributions in DCA microcellular systems exhibit very different properties to those found in FCA macrocell systems. Computer simulations and mathematical analysis have shown that DCA systems exhibit significantly closer cochannel reuse than FCA systems for a significant proportion of terminals. This is a fundamental consequence of the lack of channel partitioning in DCA.

Theoretical analysis has provided closed form expressions for the CRR distributions in DCA microcellular systems with an in–cell cochannel reuse constraint. This constraint improves the accuracy of the theoretical CRR distributions when compared with Monte Carlo simulation results. It has been shown that the resultant DCA CRR distributions cannot be obtained using macrocell principles.

These results provide a better basis for predicting the probability of the closest approach of cochannel and adjacent channel interferers, enabling better characterisation of the worst case *s*/*i* performance in DCA microcell systems.

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