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INFLUENCE OF TERMINAL DISTRIBUTIONS AND CHANNEL SPILLS ON MICROCELL COVERAGE

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Abstract – The distribution functions of cell radii in a microcell system are derived in closed form for three portable terminal distributions and tested using a Monte Carlo simulation. These distributions provide information about the quality of the microcell coverage. The results suggest that cell size reduction from near–far or same–cell interference from terminals using well spaced channels can be as severe as that resulting from adjacent channel interference from terminals in nearby microcells.

I. Introduction

Microcellular technologies are being developed to provide wireless communications to very large numbers of people at a much higher user density than is possible with conventional cellular systems [1].

Ubiquitous deployment of a high quality wireless communication system will require engineering techniques that facilitate rapid, low cost, and accurate system design [2]–[3]. As the number of deployed microcells increases, site–by–site engineering may become too time consuming and costly.

The fundamental microcell challenge is to model the end result of multiple users transmitting in a congested area [2]–[3]. Service quality targets, including cell coverage and call blocking and dropping probabilities, will need to be able to be predicted and met whilst trading off competing requirements among minimising the number of cell sites, minimising the system roll–out time, and minimising the system design cost.

Previous papers [4]–[7] have presented a general microcell interference model and analysed microcell coverage performance. This paper presents an analysis of microcell coverage performance in terms of user terminal distributions and different transmitter channel spill assumptions.

II. General Microcellular Interference Model

The general microcell interference model [4] considered an arbitrary network consisting of a fixed station F_0 and a mobile station M_{00} attempting to establish a link in the presence of n additional fixed stations F_i $\{1 \le i \le n\}$ where each fixed station communicates with a number of additional mobile stations M_{ij} $\{1 \le i \le n, 1 \le j \le m_i\}$ where m_i is the total number of mobile stations communicating with fixed station F_i as shown in Fig. 1.

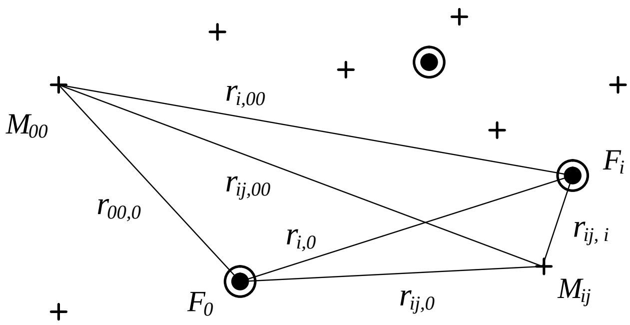

Fig. 1 Arbitrary Mobile Network

Mobile terminals transmit at a power P_{Mij} and use channels C_{Uij} on the uplink (M_{ij} to F_i) and C_{Dij} on the downlink (F_i to M_{ij}). Fixed stations transmit at a power P_{Fij} to mobile terminal M_{ij}. The notation used for distances r between transmitters and receivers is as shown in Fig. 1. This notation is independent of the multiple access method used (FDMA, TDMA, CDMA).

Channel spills from transmitter to receiver are denoted P_{XijYkl} with X_{ij} indicating the transmitter and Y_{kl} the receiver. All channel spills are incorporated regardless of their magnitude at the source (i.e. all cochannel, immediately adjacent, and further channel spills are incorporated).

For analytical tractability a single exponent distance–dependent propagation model $P_{rx} = \kappa P_{tx}(d/d_0)^{-\gamma}$ was used in [4]–[7] where γ is the path loss exponent and κP_{tx} is the received power at the reference distance d_0. However, other propagation models can be substituted.

At a receiver, a link is considered successful if the signal to noise plus interference ratio $S/[N+I]$ is greater than or equal to the system protection ratio Z, otherwise an outage is deemed to occur. In the presence of an arbitrary number of interferers it can be shown [4] that the maximum possible range $r_{00,0}$ between a mobile terminal M_{00} and its fixed station F_0 is given by:

$$r^{\gamma}_{00,0} = \psi_{M_{00}} \left[\frac{1}{\eta_{F_{00}} + 1} \right] \tag{1}$$

where $\psi_{M00} = \kappa P_{M00}/ZN$ and η_F is the uplink 'Interference to Noise Ratio' or INR. The INR at a receiver is total interference power at the receiver input divided by the receiver noise. The INR (η_F) and cell size (r) statistics can be estimated via a Monte Carlo simulation or sometimes computed analytically.

III. Cell Size Statistics as a function of Terminal Distribution

The statistics of η_F are dependent upon the statistics of the spill powers (P_{FijF00} and P_{MijF00}), $r_{i,0}$ (a constant for each i) and $r_{ij,0}$. The probability density (PDF) and distribution (CDF) functions of $r_{ij,0}$ can be computed exactly for some mobile terminal distributions. The cell radius and η_F density and distribution functions can then be derived using probability transformations.

A. Non–Roaming Terminal Distribution Model

A non–roaming terminal distribution model was presented in [6]. In this model, mobile terminals are randomly placed with a uniform distribution within the range r of a nominated server (Fig. 2). Terminals are then permitted to access that server only. This model can demonstrate the effects of *near–far* interference.

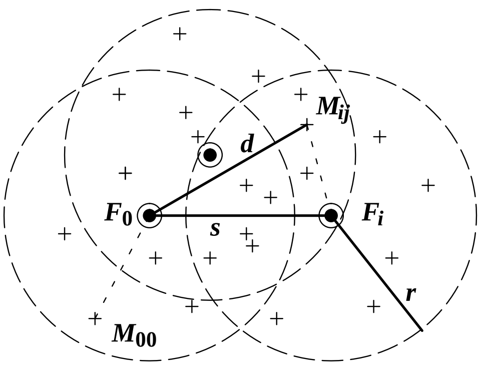

Fig. 2 Non–Roaming terminal microcell distribution model

Consider the simplest case of a two cell, two terminal system. In this case the interference model notation can be simplified as follows: $r_{i,0} = s$, $r_{ij,0} = d$, $\psi_{M00} = \psi$, and if the two terminal transmissions are sufficiently orthogonal, the channel spills $P_{FijF00} = P_{FF}$ and $P_{MijF00} = P_{MF}$ can be assumed to be constants [6]. This was denoted the 'Equal Spill Theory' (EqS).

The density function of M_{00}'s uplink INR, $f_H(\eta_F)$, was derived for the two cell, two terminal case in [6], but the density and distribution functions of cell radii were not. Denoting the maximum cell radius for M_{00} as ρ and the random variable to which this value belongs P, it can be shown that when $r \le s$ the distribution function $F_P(\rho)$ is given by:

$$F_P(\rho) = 1 - \frac{1}{\pi r^2}\left\{ s\Lambda_\rho \sqrt{1 - \left(\frac{\Lambda_\rho^2 + s^2 - r^2}{2s\Lambda_\rho}\right)^2} - \Lambda_\rho^2 \arccos\left[\frac{\Lambda_\rho^2 + s^2 - r^2}{2s\Lambda_\rho}\right] + r^2 \arccos\left[\frac{\Lambda_\rho^2 - s^2 - r^2}{2sr}\right]\right\} \tag{2}$$

where

$$\Lambda_\rho = \left\{\frac{1}{P_{MF}}\left[\frac{N}{\kappa}\left(\psi\rho^{-\gamma} - 1\right) - P_{FF}s^{-\gamma}\right]\right\}^{-1/\gamma} \tag{3}$$

When $r > s$ the distribution function becomes piecewise continuous. For $\rho > \rho_p$ the CDF of ρ is given by (2), where ρ_p is:

$$\rho_p = \left\{\frac{\psi}{1 + \frac{\kappa}{N}\left[P_{FF}s^{-\gamma} + P_{MF}(r - s)^{-\gamma}\right]}\right\}^{1/\gamma} \tag{4}$$

and when $r > s$ and $\rho \le \rho_p$ the CDF of ρ is given by:

$$F_P(\rho) = \frac{\Lambda_\rho^2}{r^2} \tag{5}$$

This result will be called the Equal–Spill Non–Roaming Theory (EqS–NR).

B. Constrained Non–Roaming Terminal Distribution Model

In this model, mobile terminals are distributed as in the previous model, except a constraint is applied that a terminal's nominated server must be the closest server. If this is not the case, that terminal is not admitted and is replaced.

By restricting terminals to their closest server (usually done in practice on the basis of received signal strength indication – RSSI) the *near–far* problem of cell overlap is eliminated, and interference should be greatly reduced.

Consider again the two cell, two terminal case under the equal spill assumption. If $r \le s/2$ the cells are discrete and the distribution function $F_P(\rho)$ is given by (2). If $r > s/2$ the cells touch and the distribution function $F_P(\rho)$ becomes piecewise continuous. It can be shown that when $r > s/2$ the distribution function $F_P(\rho)$ is given by:

$$F_P(\rho) = \begin{cases} \dfrac{\Lambda_\rho^2 \arccos\left(\frac{s}{2\Lambda_\rho}\right) - \frac{s}{2}\sqrt{\Lambda_\rho^2 - \frac{s^2}{4}}}{\frac{s}{2}\sqrt{r^2 - \frac{s^2}{4}} + r^2\left[\pi - \arccos\left(\frac{s}{2r}\right)\right]} & \rho \le \rho_q \\[2ex] \dfrac{\pi r^2 - s\Lambda_\rho\sqrt{1 - \left(\frac{\Lambda_\rho^2 + s^2 - r^2}{2s\Lambda_\rho}\right)^2} + \Lambda_\rho^2 \arccos\left[\frac{\Lambda_\rho^2 + s^2 - r^2}{2s\Lambda_\rho}\right] - r^2 \arccos\left[\frac{\Lambda_\rho^2 - s^2 - r^2}{2sr}\right] + \frac{s}{2}\sqrt{r^2 - \frac{s^2}{4}} - r^2\arccos\left(\frac{s}{2r}\right)}{\frac{s}{2}\sqrt{r^2 - \frac{s^2}{4}} + r^2\left[\pi - \arccos\left(\frac{s}{2r}\right)\right]} & \rho > \rho_q \end{cases} \tag{6}$$

where Λ_ρ is given by (3) and ρ_q is given by:

$$\rho_q = \left\{\frac{\psi}{1 + \frac{\kappa}{N}[P_{FF}s^{-\gamma} + P_{MF}r^{-\gamma}]}\right\}^{1/\gamma} \tag{7}$$

This result will be called the Equal–Spill Constrained Non–Roaming Theory (EqS–CNR).

C. Roaming Terminal Distribution Model

Now consider a terminal distribution model with full inter–operator roaming. User terminals are placed randomly throughout the microcell *service area* with a uniform distribution and each mobile terminal chooses the 'best' server on the basis of RSSI.

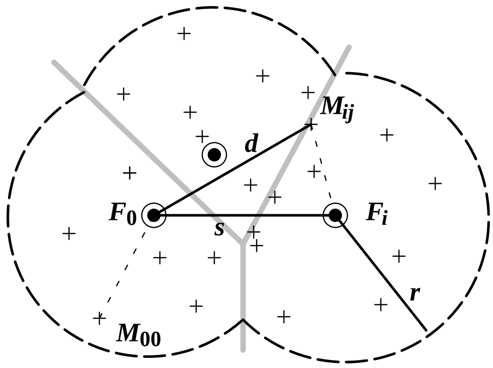

Fig. 3 Roaming terminal microcell distribution model

The grey lines in Fig. 3 divide the service area into three regions where portable terminals will only access the server in that region. Clearly, this model can result in some servers handling more traffic than others.

Again consider the two cell, two terminal case under the equal spill assumption. The set of possible server admissions $\mathcal{A}$ is divided into two mutually exclusive events:

$\mathfrak{S}$: The terminals choose the same server
$\mathfrak{D}$: The terminals choose different servers

The distribution function $F_P(\rho)$ can thus be computed for this model using the total probability theorem:

$$F_P(\rho) = F_P(\rho|\mathcal{A}) = F_P(\rho|\mathfrak{S})P(\mathfrak{S}) + F_P(\rho|\mathfrak{D})P(\mathfrak{D}) \tag{8}$$

The distribution $F_P(\rho|\mathfrak{D})$ is the Constrained Non Roaming Terminal distribution given by (6). The distribution $F_P(\rho|\mathfrak{S})$ is that resulting from same–cell interference. It can be shown that when $r > s/2$ and $\rho \le \rho_r$ the distribution $F_P(\rho|\mathfrak{S})$ is given by:

$$F_P(\rho|\mathfrak{S}) = \frac{\pi\zeta_\rho^2}{\frac{s}{2}\sqrt{r^2 - \frac{s^2}{4}} + r^2\left[\pi - \arccos\left(\frac{s}{2r}\right)\right]} \tag{9}$$

and when $r > s/2$ and $\rho_r < \rho \le \rho_s$:

$$F_P(\rho|\mathfrak{S}) = \frac{\frac{s}{2}\sqrt{\zeta_\rho^2 - \frac{s^2}{4}} + \zeta_\rho^2\left[\pi - \arccos\left(\frac{s}{2\zeta_\rho}\right)\right]}{\frac{s}{2}\sqrt{r^2 - \frac{s^2}{4}} + r^2\left[\pi - \arccos\left(\frac{s}{2r}\right)\right]} \tag{10}$$

where:

$$\zeta_\rho = \left\{\frac{1}{P_{MF}}\left[\frac{N}{\kappa}\left(\psi\rho^{-\gamma} - 1\right)\right]\right\}^{-1/\gamma} \tag{11}$$

$$\rho_r = \left\{\frac{\psi}{1 + \frac{\kappa}{N}[2^\gamma P_{MF}s^{-\gamma}]}\right\}^{1/\gamma} \tag{12}$$

$$\rho_s = \left\{\frac{\psi}{1 + \frac{\kappa}{N}[P_{MF}r^{-\gamma}]}\right\}^{1/\gamma} \tag{13}$$

Due to the symmetry of the two cell, two terminal system, clearly $P(\mathfrak{S}) = P(\mathfrak{D}) = 0.5$ thus $F_P(\rho) = 0.5[F_P(\rho|\mathfrak{S}) + F_P(\rho|\mathfrak{D})]$. The result will be called the Equal–Spill Roaming Theory (EqS–R).

D. Comparison Between the Terminal Distribution Models

The three terminal distribution models may be compared by plotting their distribution functions for a specific case. Consider a two cell, two terminal CT2 system with the parameters as shown in Table 1.

TABLE 1 – CT2 SYSTEM PARAMETERS

Parameter	Value
γ	3.0
κ	–11.2 dB
N	–111.0 dBm
r	100 m
s	100 m

Figs. 4 to 6 show the resultant INR density $f_H(\eta_F)$, INR distribution $F_H(\eta_F)$, and cell radius distribution $F_P(\rho)$ functions respectively for the three terminal distribution models.

Figs. 4 and 5 illustrate that all terminal distribution models have the same lower INR bound as a consequence of the cell layout geometry, but not the same upper bounds. The EqS–R INR density function exhibits a step at the lowest INR value at which it is possible for the two terminals to choose the same server. This step appears as a kink in the INR distribution graph of Fig. 5.

The INR and cell radius distribution functions (Figs. 5 and 6) are plotted on normal and logarithmic probability scales respectively to amplify the tails of the distributions at high INRs and small cell radii. This enables the proportion of terminals experiencing below target cell sizes to be highlighted.

The EqS–CNR model leads to a very small spread of cell radii (Fig. 6) as neither near–far nor same–cell interference is possible. However, the cell radii distributions for the other two models are similar, suggesting that *same–cell* interference can be as deleterious to system performance as *near–far* interference.

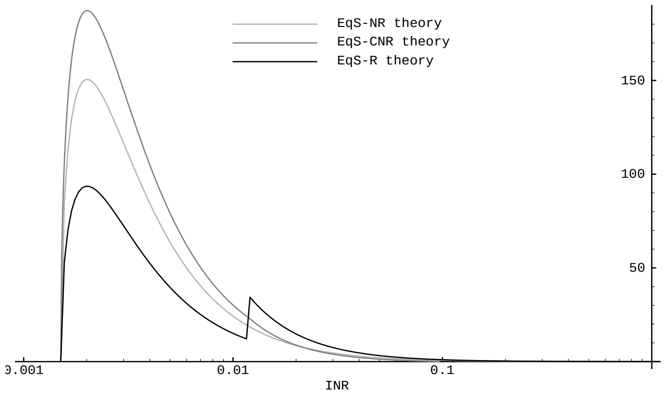

Fig. 4 $f_H(\eta_F)$ **for the three terminal distribution models**

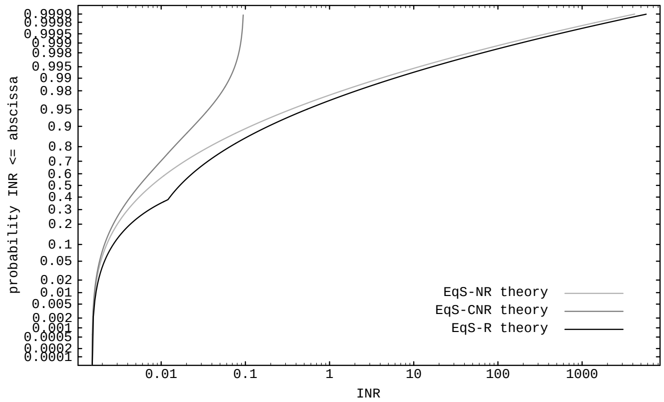

Fig. 5 $F_H(\eta_F)$ **for the three terminal distribution models**

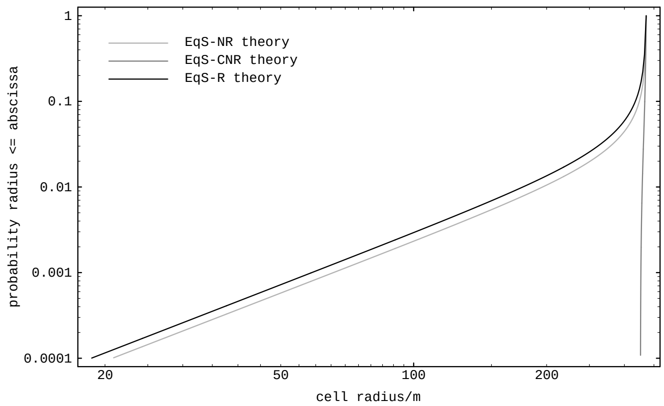

Fig. 6 $F_P(\rho)$ **for the three terminal distribution models**

IV. Monte Carlo Evaluation of Cell Coverage

The theoretical cell size distribution results can be tested using a microcell interference simulation program [4]–[7] with mobile terminals placed in accordance with the terminal distribution models as described in section III.

A two cell, two terminal CT2 network with simulation parameters as per Table 1 was implemented. The CT2 base stations were assumed to be synchronised ($P_{FF} = 0$) and power control was not used. Two simulations were then performed for each terminal distribution model, with cell radius statistics for the *successful calls* collected from 10000 random call attempts for each simulation condition.

The first simulation for each terminal distribution model applied the equal–spill assumption by constraining terminal transmissions to be at least 3 RF channels apart, giving a constant P_{MF} of –39 dBm. These simulations should agree closely with the EqS theory.

The second simulation then allowed full DCA in accordance with the CT2 specification, hence P_{MF} varied in accordance with the actual channel allocations made at call set–up time (ETSI channel spills were used [8]). Simulations using full DCA are denoted 'exact spill' (ExS) simulations and test the accuracy of the equal spill simplification.

Fig. 7 shows the results for the Non–Roaming terminal distribution. The EqS–NR simulation follows the EqS–NR theory almost exactly except for the last percentile of the distribution. This divergence is caused by the Monte Carlo simulation clearing calls which fail to meet the required S/[N+I]. Calls are more likely to be blocked or dropped if their INR is high or their maximum range is small. Hence the cell radius distribution for successful calls is skewed away from very small cell radii.

The ExS simulation compares reasonably well with the EqS theory save for the 1–5 percent region. With full DCA, it is possible for cochannel and immediately–adjacent channel interference to be generated. This increases interference and the probability of smaller cell radii until the S/[N+I] constraint for a successful call comes into play. In all cases, it can be seen that near–far interference causes significant reductions in cell size.

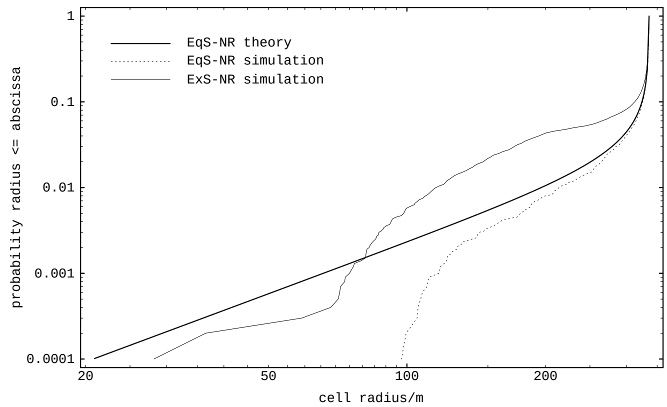

Fig. 7 $F_P(\rho)$ **for Non–Roaming terminal distribution**

The results for the Constrained Non–Roaming terminal distribution are shown in Figs. 8 and 9 (Fig. 9 expands the last part of Fig. 8). The agreement between the EqS–CNR simulation theory is excellent as no call attempts failed in this simulation. The absence of near–far interference coupled with well spaced RF channels leads to a very small range of cell sizes as expected.

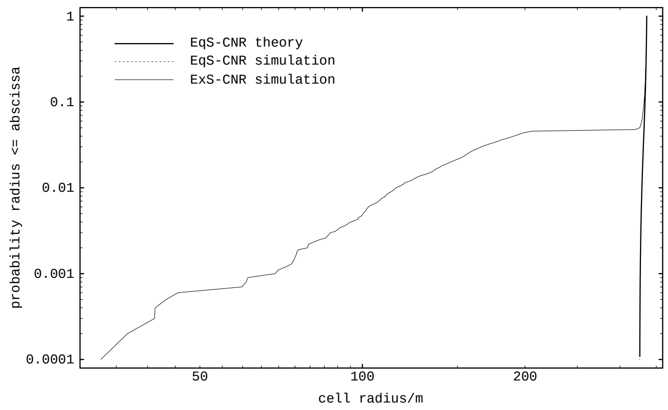

Fig. 8 $F_P(\rho)$ for Constrained Non–Roaming terminal distribution

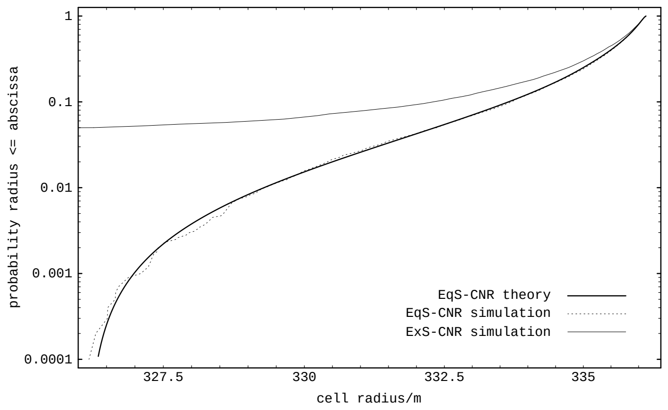

Fig. 9 $F_P(\rho)$ – Expanded part of Fig. 8

However, the ExS simulation bears little resemblance to the EqS theory. This result suggests that a terminal in *another cell* choosing nearby channels under DCA can affect the system performance almost to the same degree as a *near–far* interferer choosing a well–spaced channel.

Further, the results of the roaming terminal distribution model (Fig. 10) suggest that enabling roaming does not solve either problem. Roaming eliminates near–far interference, but the results in Fig. 10 indicate that well spaced channels (EqS simulation) and DCA (ExS simulation) both cause same–cell interference, resulting in a significant reduction in cell radius to a significant proportion of terminals.

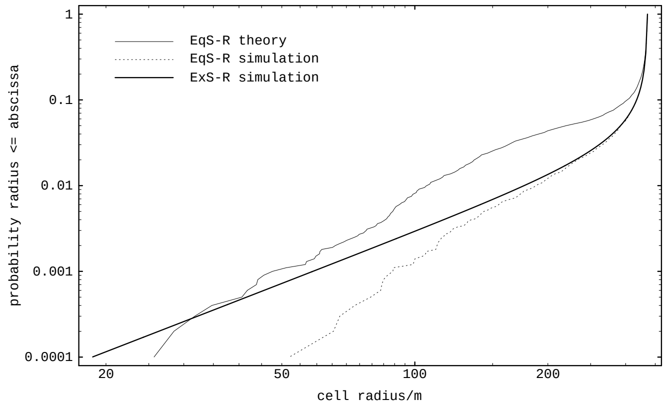

Fig. 10 $F_P(\rho)$ for Roaming terminal distribution

The other interesting aspect of Fig. 10 is that the difference between the EqS and ExS simulations is not as great as in the previous two terminal distribution models. This suggests that the DCA algorithm is reasonably effective in ensuring that terminals use well spaced channels, but it also suggests that even in the absence of cochannel and immediately adjacent channel interferers, same–cell interference effects are significant.

As the user density in a microcell system increases, such interference effects can only increase. Under high user density conditions it may not be sufficient to control microcell interference by using DCA and adopting strategies to combat the near/far problem.

V. Conclusion

The distribution functions of cell radii in a microcell system were derived in closed form for three portable terminal distributions and tested using a Monte Carlo simulation. The results suggest that cell size reduction from near–far or same–cell interference from terminals using well spaced channels can be as severe as that resulting from adjacent channel interference from terminals in nearby microcells.

VI. References

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